Understanding and Using Fourier Transform Thin Film Design Without Advanced Calculus

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ABSTRACT

A desired reflectance profile versus frequency (wavenumber) can be directly Fourier transformed into the index of refraction profile as a function of thickness which would produce the desired spectral reflectance profile. This tool has existed for some years now but does not seem to be in common use. This paper lays out some of the principles to aid in making the process more understandable, intuitive, and useful without recourse to higher mathematics. If nothing else, the Fourier transform technique can provide a good starting point for further design optimization with commonly used techniques. It can be very valuable for new and unusual thin film design problems for which previous experience is of little guidance. It can also add insight into underlying principles of common designs of longstanding.

INTRODUCTION

Those of us who design optical coatings have generally become skilled at guessing what might be a good starting point for a design to meet a given requirement. We then optimize this starting design with respect to the design objectives using some appropriate thin film design software. In many cases, this approach leads to satisfactory solutions. Experience and understanding are usually the major factors in determining how quickly one solves the problem at hand. However, in principle, it is possible to specify the spectral reflectance required and directly synthesize a layer structure which will meet the requirements. The basic tool to do this is the Fourier technique described by Delano[1], Sossi[2], Dobrowolski[3], Bovard[4], and Southwell[5]. A study of these papers might tend to dampen the enthusiasm of those less mathematically inclined. Also, most of us have managed to get along well enough without these tools thus far. However, we have encountered problems where intuition and experience do not indicate how to best design for the creation of some complex reflectance profiles. In these cases, it would be highly advantageous to have a tool to take the required reflectance profile and synthesize a solution which would meet the requirements. Within certain reasonable limitations, this is now possible with the Fourier techniques, but they are not well understood or widely available at this time. It is the purpose of this paper to help make the techniques more readily understood so that they will be useful as they become more available. The published works to date have been heavy in mathematical description. We will avoid the math and focus on the concepts with graphical illustration. The math may be the engine that propels this vehicle, but we need not be an engine designer or an auto mechanic in order to drive the car to where we want to go. We try to present here a “Driver’s Ed” version of what makes the “car go”.

BACKGROUND (HOW IT WORKS)

We all work with the fact that there is a Fresnel reflection which occurs at an interface between two different refractive indices such as air and glass. This is simply described by:

r=(n0-n1)/(n0+n1)

where r is the reflectance amplitude and n0 and n1 are the refractive indices on either side of the interface. The reflected intensity (R) as we measure it is the square of r (R=r*r). The reflectance of this single interface is the same for all wavelengths (or frequencies) if there is no change of indices with frequency (no dispersion). Note also that there are no reflections if there is no change in index. Figure la shows index of refraction versus position (t= optical thickness) in space along the direction of the propagation of the light. Figure 1b shows the reflectance amplitude (r) versus position, and Fig. 1c shows the reflectance intensity (R=r*r) versus frequency (f=1/wavelength).

Now if we take the case of a thin slab of material surrounded by a lower index medium, we know that we get interference between the reflections from the first and second surfaces. Figure 2a shows the index profile of a slab of index 1.5 in a vacuum (or air), Fig. 2b shows the reflectance (r) versus thickness. Figure 2c shows the reflectance intensity (R) versus frequency. This is the familiar result for the single layer coating. If we double the thickness of the slab as shown in Fig. 3, the frequency doubles. If we tripled the thickness, the frequency would triple, etc., etc., etc.

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If we now space two of the previous slabs apart by an optical thickness equal to their own, we get the results seen in Fig. 4 where interactions of the various reflections add in relative frequencies and phases to produce the results shown. Note that the R scale has quadrupled because the multiple reflections have added up to 2X greater to give 2x2 the reflection at the peak.

If we analyze the common quarter wave optical thickness (QWOT) stack as in Fig. 5, we see a familiar result where there is a reflectance peak at the fundamental frequency of the QWOT stack and each odd harmonic of that frequency. This is because the even harmonics are where the stack would be half waves and therefore absentee layers or non-reflectors. Figure 6 shows the result of changing the relative thicknesses of the high and low index layers to a 2:1 ratio rather than the 1:1 of the QWOT stack. We see that the reflectance of the peaks change to include the second and forth harmonic and suppress the third. This technique, to control wanted and unwanted harmonic peaks, was described by Baumeister[6] and has found broad use in recent years for laser blocking filters.

Figure 7 illustrates variable indices and thicknesses. This is the classical quarter-half-quarter wave three layer broad band antireflection coating on crown glass. It can be seen that the broad AR band is flanked on each side by moderately high reflection bands (which are seldom discussed or displayed).

What can we see from all of the above? One thing is that the reflectances at interfaces add in amplitude and phase to give a reflection profile as a function of frequency. More widely spaced interface reflections give higher frequency effects and narrower spacings give lower frequency effects. The lowest frequency is zero (0) as in the single interface case. This causes a "DC" shift of the resultant sum. It is also fairly apparent that higher interface reflectances contribute to higher reflectances as a function of frequency.

What does all of this have to do with Fourier techniques? The resulting reflectances versus frequency are nothing but the Fourier Transform of the reflectance profiles versus position or thickness! We could get the same result, of course, by the usual matrix calculations of reflectance of multilayers when given the indices and optical thicknesses of the layers. The BIG DIFFERENCE is that the Fourier Transformer works both ways. It is the same as an AC voltage transformer that can step up 110V to 220V or can also be reversed to step down 220V to 110V. This means that the Fourier Transformer can take a reflectance versus frequency profile and transform it to a reflectance versus optical thickness profile just as well as it can transform the index profile to a spectrum. This can then be converted to an index vesus thickness profile, which is the solution we wanted. In principle (and in the extreme) then, we could expect to directly calculate the multilayer stack which would produce the required reflectance. No "design" talent or optimization would be required.

The Fourier technique, therefore, holds the promise of being an additional tool to help develop thin film designs to meet requirements of reflectance profiles whose best solutions are obscure to the designer. We would expect to see this tool used more often in the future as a "hammer" to crack the "tough nuts" of thin film design problems.

CURRENT LIMITATIONS

There are, of course, various limitations to be aware of in the current Fourier technology. We can really only work at this time with non-dispersive and non-absorbing materials. As Dobrowolski[3] points out in detail, the choice of phase is not determined by the desired reflectance profile, so there are an infinite number of theoretical answers to a given problem. The problem then is to select solutions that are practical in the sense of using available and desirable materials and having realistic thicknesses. Dobrowolski, et al.[3] have gone into these issues is some detail. Bovard[4] has reported other supporting studies. There seems to be many areas of this technology yet to be investigated, fully understood, and applied.

Dobrowolski's system currently seems to combine the synthesis capability of the Fourier techniques with some optimization techniques to overcome some of the current limitations of the Fourier techniques.

Dobrowolski and Bovard have both mentioned the approximations of the "Q-functions" which have been used to date and the limitations imposed by those approximations. Q-functions are the relationship of the amplitude in index of refraction versus optical thickness domain to the reflectance versus frequency domain. Bovard also discusses the apparent frequency scale distortions (in previous Fourier techniques) around high reflection bands and how this might be approximately corrected. The two problems of the Q-function and the apparent frequency scale distortions have interested us for some time as a result of an earlier collaboration with Verly and Dobrowolski (7) and our own subsequent investigations.

RESULTS

We will briefly summarize the findings of our investigations to date which we plan to report on in more detail at the International Thin Film Conference in Tucson in June. One facet of our work has been the search for a single universally applicable Q-function which can be applied to both high and low reflectance cases equally well. The second aspect has been an attempt to resolve the frequency distortion surrounding high reflection peaks such as QWOT stacks. Our approach has been empirical in nature rather than strictly mathematical. We have taken the results of accepted matrix solutions and transformed them to generate the functions which would, when transformed, produce the established results. We then studied these results to find indications of the nature of the relationships of index profile, multiple reflections, and reflection versus frequency. Our work to date has mostly been directed at gaining understanding of the underlying principles and analysis as a precursor to the ultimate systhesis system desired.

We are now convinced of two facts as a result of our studies. The first is that a Q-function that applies to both high and low reflection cases universally is the reflectance amplitude itself as described above. This can be seen by the fact that each interface reflection adds its amplitude to the reflectance versus frequency result and at a unique frequency and phase determined by its position with respect to the other reflectances in the stack. We have seen no reason why any other Q-function would be more correct, useful, or basically simple. This was not clear to us from our study of the literature.

The second conclusion that we have come to is that there are no distortions in the reflectance versus frequency result if the multiple reflections between interfaces are properly taken into account. This also was not clear to us from the literature. Figure 8b shows the reflectance of the interfaces of a germanium slab of index 4 in a vacuum and the multiple internal reflections. Figure 8c shows the resulting Fourier transform squared, which agrees in detail with the matrix calculation of the same “design.” As more interfaces than just two are added to the problem, the interactions of each interface with each other must be included. We have yet to find the proper approach to doing this. We suspect that the solutions to this will be related to the recursive relationships whereby multiple reflections are accounted for in reflectance calculations as described by Heavens[8]. If this problem can be solved, then both the Q-function question and the apparent “distortion” with frequency problem will not only have been explained, but may be carried forward to true synthesis of optical thin film designs.

SUMMARY

We have illustrated how the reflectance versus optical thickness of a thin film structure transforms to the reflectance versus optical frequency of that structure. We have reported that the Q-function question is answered by the fact that reflectance transforms to reflectance! We have demonstrated that there is no frequency distortion when multiple reflections between layers are properly taken into account. We expect that Fourier synthesis will become a more widely used tool in the future as some of the current limitations are overcome.

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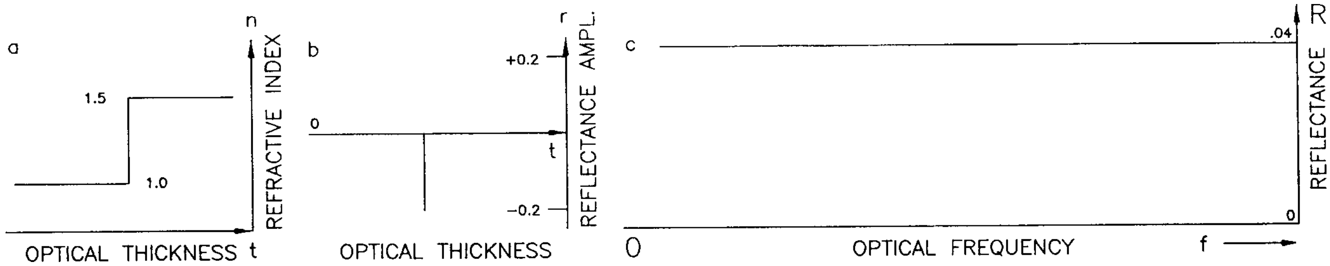

Fig. 1 Single interface between glass and air (vacuum), seen in index profile, reflectance amplitude profile, and reflectance intensity (R=rxr) versus frequency (1/wavelength).

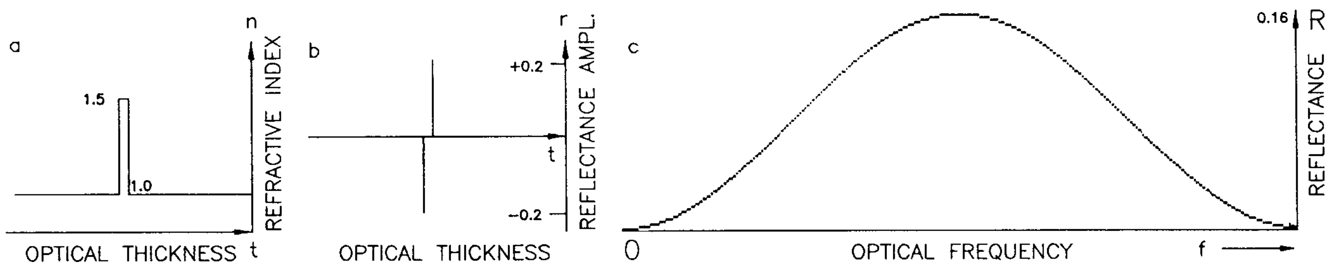

Fig. 2 Two interface glass slab (thin) in air, displayed as in Fig. 1.

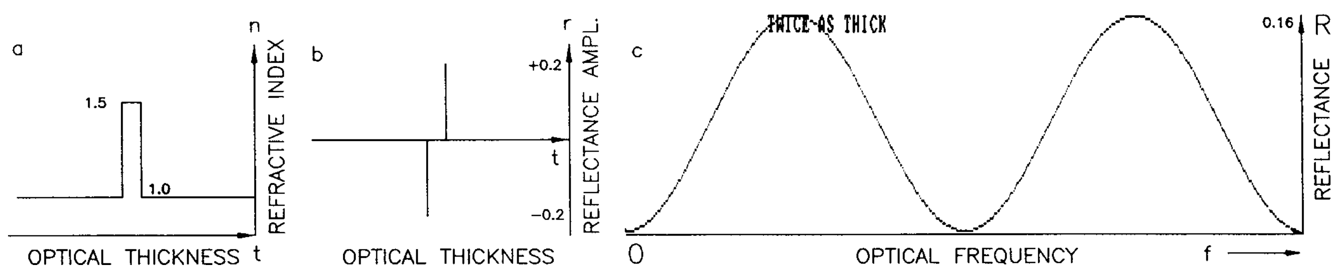

Fig. 3 Two interface glass slab twice as thick as in Fig. 2.

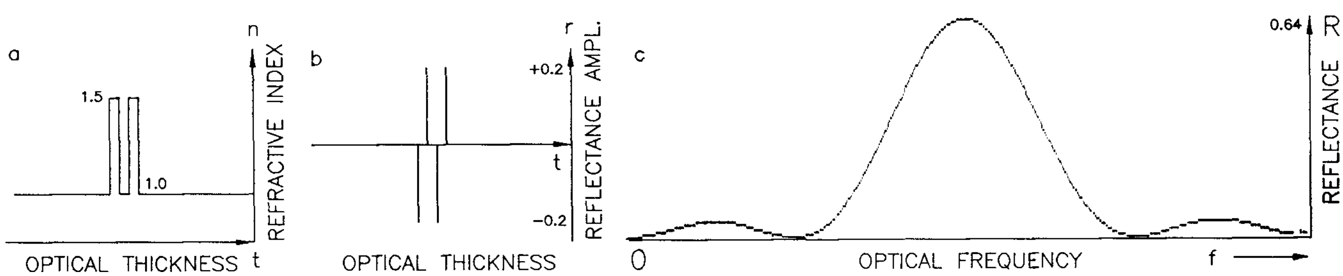

Fig. 4 Four interfaces, two slabs equally spaced or a three QWOT stack.

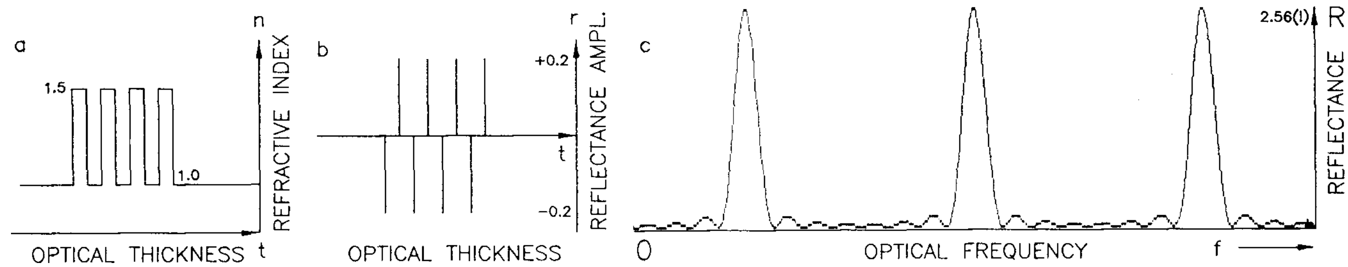

Fig. 5 Eight (8) interfaces or seven (7) QWOT stack. The R > 1.0 shows the error caused by not correctly accounting for the multiple reflections as is done in the Germanium case of Fig. 8.

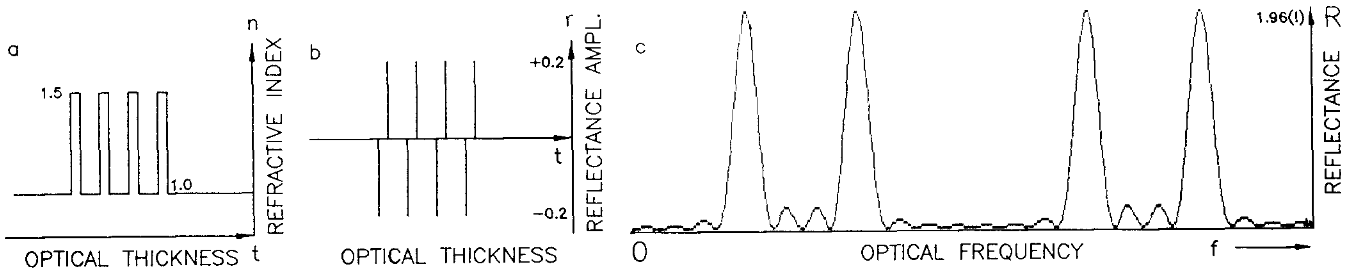

Fig. 6 2:1 thickness ratio stack of eight interfaces. Compare to Fig. 5 which has a 1:1 ratio.

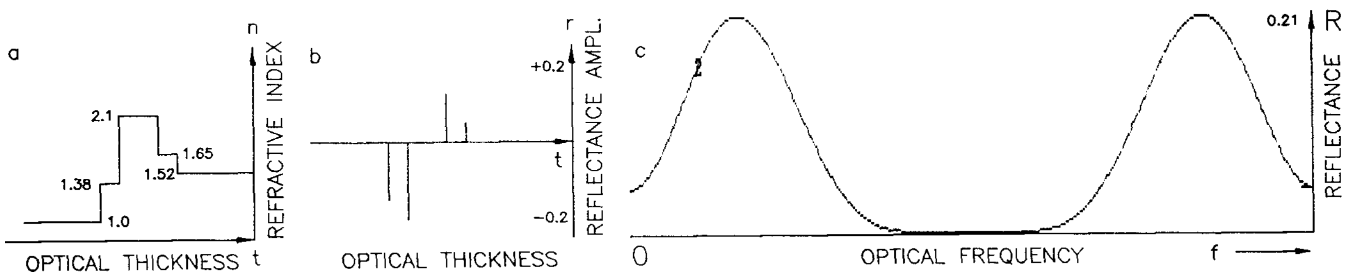

Fig. 7 Classical three layer antireflection coating on crown glass.

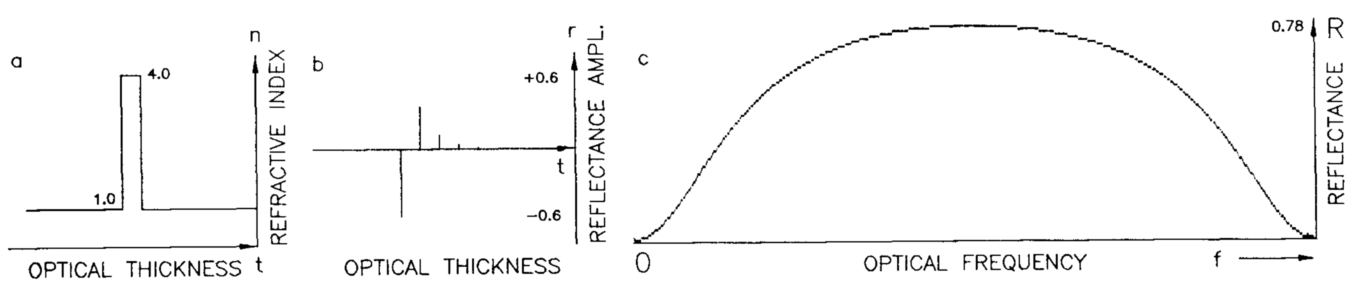

Fig. 8 Germanium slab in air with multiple reflections properly taken into account. This agrees in detail with matrix calculation results.