Machine-Aided Selection of Optical Glasses for Two-Element, Three-Color Achromats

R. R. Willey, Jr.

Research Laboratories, United Aircraft Corporation, East Hartford, Conn.
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The work of Herzberger[1] and Stephens[2] dealing with the necessary conditions and optical glass selection needed to achieve four-color achromats stimulated our interest in seeking combinations of glasses that would give four-color correction for longitudinal chromatic aberration. We have not been successful in finding such a pair of glasses, but we have as a result found several glass pairs which give three-color correction. These may prove to be of some value to astronomers. The general methods used to find these combinations are discussed herein.

The methods that we have used are, in principle, those of Stephens[2] but we have adapted them for use in our high-speed digital computers (IBM 7090, Philco 2000).

The first step (see Fig. 1) is to transfer the index of refraction data from the glass manufacturers' catalogs to punched cards. The identification of the glass and manufacturer are put on the card, and weathering, acid, availability, etc., codes can be included as desired. This process need be done only once, except where errors need to be corrected

Secondly, the index data are fitted by the method of least squares to Herzberger's "Universal Function" formula to minimize the effects of measured and typographical errors in the glass tables. The form of the dispersion formula currently used is shown at the bottom of Fig. 1. This program produces a deck of cards which constitutes a "Universal Glass Catalog." Each card of the deck has the four fitted indices for the glass at the A', C, F, and h Fraunhofer lines of the spectrum and the identification of the manufacturer and glass type. The durability codes may be included, and additional comments can be added to the cards at a later date as desired. This fitting step need be done only once also, unless it is desired to fit data to some other formula such as has been done by Sutton and Stavroudis.[3]

The third step is to compute from these fitted indices the reciprocal dispersion and whatever partial dispersions are desired. Since the interpolation formula above is used for finding the index at any wavelength, the dispersions can be found for any wavelength within the region of validity of the particular interpolation formula used. This fact should prove to be of value since the best partials to be examined will depend on the wavelength region in which the residual spectrum is to be minimized. If the region from C to F light is not included in the region of
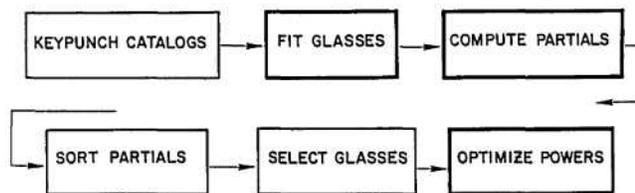
Fig. 1. The steps in the procedure for selecting two-element, three-color achromats. The steps in heavier outline are currently done on a high-speed computer (IBM 7090). The dispersion formula currently used in the glass fitting operation is shown below the block diagram.
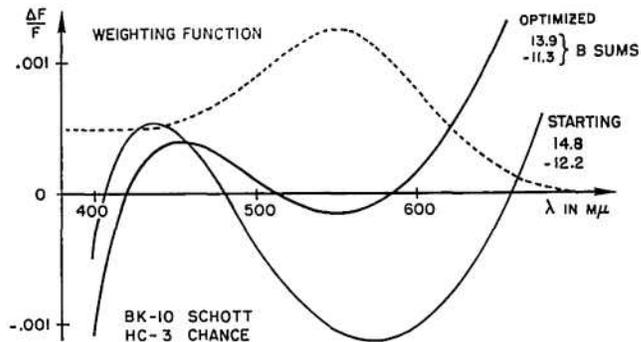
Fig. 2. The normalized change in focal length is plotted as a function of wavelength for a pair of glasses before and after being optimized with respect to the weighting function shown.

primary interest, a reciprocal dispersion based on wavelengths in the region of interest should lead to better results. This arises because the relative powers of the elements depend on dispersion which is a function of wavelength and, therefore, the region to be achromatized.

The computed dispersions are punched out by the computer with the same identification data that appear on the index catalog cards. The data from these cards could then be plotted on the three view graphs that Stephens described for the selection of three- or four-element, four-color achromats. However, if one is searching only for two-element combinations, the problem becomes well suited to the facilities of a modern computing laboratory.

The cards are sorted, in the fourth step, by an off line card sorter which puts one of the partial dispersions in ascending order. The ordered deck is then listed out by an off line printer. Table 1 shows a small sample of such a listing.

Table 1. A Few Lines from the List of Dispersions Ordered with Respect to the e-C Partials[a]

$\nu = \dfrac{n_d - 1}{n_F - n_C}$	$P = \dfrac{n_h - n_C}{n_F - n_C}$	$Q = \dfrac{n_{1 \cdot 014} - n_C}{n_F - n_C}$	$R = \dfrac{n_e - n_C}{n_E - n_C}$	Identification
63.386	1.9775	0.85542	0.544502	Chance LBC-1
63.972	1.9771	0.85945	0.544694	Chance BSC-3
67.325	1.9797	0.86792	0.544796	Schott FK-6
50.677	2.0185	0.95544	0.544840	Hayward CF-2
64.273	1.9729	0.85559	0.544909	Chance BSC-1
64.273	1.9782	0.87167	0.545066	Schott BK-7

[a] The definitions of the dispersions are shown above each column.

The fifth step currently utilizes the designer, although this operation could be programmed if it were deemed advisable. One scans the listing to find glasses whose reciprocal dispersions are sufficiently different to give a practical combination and whose partial dispersions are nearly the same, so that these glasses will give rise to a reduced secondary spectrum. If a pair of glasses with significantly different reciprocal dispersions has identical partial dispersions for one of the partials, the thin lens combinations of these glasses would be corrected for three colors. The difference in the partials indicates the residuals of color at those wavelengths for which the partials were computed. A pair of glasses may provide three-color correction even though the particular partials computed are not identical; some other wavelengths may be corrected. However, if the differences in

the computed partials are large, the correction for three colors will be poor (even if it exists) because of residuals between the corrected wavelengths.

The sixth step is performed after promising pairs of glasses have been chosen by the previous step. The "Universal Catalog" index cards for these pairs of glasses are fed into a program which computes the error in focal length divided by the focal length as a function of wavelength for as many wavelengths as desired. The ratio of the powers of the two elements is then varied, and the optimal powers are found by a convergent procedure similar to that described by Meiron and Volinez.[4] Figure 2 shows an example of the residual longitudinal chromatic aberration of a glass pair before and after optimization of lens powers. A number representing the merit of the resulting combination with respect to residual spectrum is produced. The various pairs can then be compared by their merit numbers and the powers and durabilities of their elements.

Our experience to date is similar to that of earlier workers in that we have found that the durability and desirability of the glasses of greatest interest for two-element, three-color achromats are almost in inverse proportion to their ability to eliminate secondary longitudinal chromatic aberration.

References

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Abac Chart for Fast Calculation of the Absorption and Reflection Coefficients

W. Nazarewicz, P. Rolland, E. da Silva, and M. Balkanski

Laboratoire de Physique, Ecole Normale Supérieure, Paris (W. Nazarewicz is on leave from the Institute of Nuclear Research, Warsaw.)
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Simultaneous transmission and reflection measurements on a thick absorbent layer permit one to obtain the absorption and reflection coefficients. A fast calculation is proposed by using a double coordinate system abac chart.

The Drude theory[1] relates the reflection index n and the extinction coefficient k to the absorption coefficient K for a given wavelength, and the reflection coefficient r on the surface of an absorbent material by the equations:

$$r = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2}, \qquad (1)$$

$$K = \frac{4\pi k}{\lambda}. \qquad (2)$$

For an absorbent material it is convenient to make the measurements by means of a thick homogeneous layer with plane parallel faces illuminated by parallel light under normal incidence.

Under these conditions, multiple reflections are produced in the interior of the sample. The resultant intensity due to the contributions of all these reflections must be computed. Czerny[2] and Barnes and Czerny[3] have shown that the measured transmittance T and the reflectance R can be obtained by the formulas:

$$T = \frac{(1-r)^2 + 4r \sin^2 \psi}{(e^{2\pi kd/\lambda} - re^{-2\pi kd/\lambda})^2 + 4r \sin^2 [(2\pi nd/\lambda) + \psi]}, \qquad (3)$$

and

$$R = r\frac{(e^{2\pi kd/\lambda} - e^{-2\pi kd/\lambda})^2 + 4\sin^2(2\pi nd/\lambda)}{(e^{2\pi kd/\lambda} - re^{-2\pi kd/\lambda})^2 + 4r \sin^2 [(2\pi nd/\lambda) + \psi]}, \qquad (4)$$

on introducing

$$\tan\psi = \frac{2k}{n^2 + k^2 - 1}, \qquad (5)$$

where d is the thickness of the layer.

Fan and Becker[4] have reduced Eqs. (3) and (4), for the case of thick layer, by taking the average of the number of waves, d/λ to

$$T = \frac{(1-r)^2 + 4r \sin^2 \psi}{e^{Kd} - r^2 e^{-Kd}},$$

which is

$$T = \frac{(1-r)^2 e^{-Kd}}{1 - r^2 e^{-2Kd}}, \qquad (6)$$

for not very absorbent layers. Similarly, one obtains

$$R = r(1 + Te^{-Kd}). \qquad (7)$$

The formulas (6) and (7) can also be obtained directly by summing up the intensities due to the contributions of all the multiple reflections on the faces of the layer.

A calculation of the optical constants requires, first of all, the solution of Eqs. (6) and (7) in order to obtain r and K. Replacing r by its value in Eq. (6) one can solve the third-degree equation

$$(e^{-Kd})^3 + \left[\left(\frac{1-R}{T}\right)^2 - 1 + \frac{1}{T^2}\right]T(e^{-Kd})^2$$

$$+ \left[\left(\frac{1-R}{T}\right)^2 - 2\right]e^{-Kd} - \frac{1}{T} = 0. \qquad (8)$$

This equation can be solved for example by the method of Cardano. The calculations are long and painful. Besides, it is difficult to use them for the relating of systematically experimental measurements. Consequently, it is preferable to proceed by graphic solution.

The curves giving T and R for constant values of Kd and r have been traced (Fig. 1). We have taken a sufficient number of values for Kd and r in Eqs. (6) and (7) in order to permit easy interpolation.

Taking, on the abac chart, a point defined by its coordinates T and R, it is easy to read its coordinates in the system e^{-Kd}, r. Thus, Eq. (8) has been solved.

This method of graphic solution is valid only when the error introduced by the trace and the interpolation of the coordinates of points is small in comparison with the error on the experimental values of T and R. This is usually the case.

It has been concluded that the proposed abac chart permits one to obtain rapidly the transmission and the reflection coefficients of thick layers. Its usefulness is evident in the systematic study of the optical properties of such materials as semiconductors, which are neither perfectly transparent nor opaque, and which it