Graphic description of equivalent index approximations and limitations

Ronald R. Willey

We show the principles of Herpin index and Epstein equivalent period approximations in graphic form and discuss the possibilities and limitations of the approximations with changes in wavelength and angle of incidence.

I. Introduction

When a given index of refraction is required to realize a particular thin film design but no suitable material is available, it is possible to approximate the desired index material with two other available materials of index greater and less than the desired index. This concept is usually referred to as the Herpin[1] index and Epstein[2] period. It has been used and described by Berning,[3] Ohmer,[4] Macleod,[5] Liddell,[6] etc. These authors, Rabinovitch and Ziv[7], and others have also pointed out that these approximations are not entirely equivalent except at one specific wavelength and angle of incidence. Knittl and Houserkova[8] have described the use of equivalent layers at oblique incidence, and Costich[9] showed how to reduce polarization effects in coatings using equivalent index concepts. We show here in graphic form by means of admittance diagrams the principles of the Herpin index and symmetric periods attributed to Epstein, how they can be used, and how the limitations can be overcome if necessary.

II. Equivalent Layer Principles

The typical broad band antireflection coating for the visible spectrum can be done with three layers of index 1.65, 2.1, and 1.38. The reflectance spectrum of a quarter-half-quarter wave (QHQ) stack of these materials is seen in Fig. 1. At a central wavelength in the design, the admittance diagram would be as in Fig. 2. If the 1.65 material was not available or we wanted to eliminate it, it could be replaced by a combination of layers made of the 2.1 and 1.38 materials which had approximately the equivalent behavior of the 1.65 layer. Figure 3 is used to show the principle in graphic form. When a QWOT of 1.65 index is deposited on a substrate of index 1.52, the admittance of the combination moves from point A to point Z along the semicircle labeled M for medium index. Any combination of layers which brings the admittance from point A to Z will have the same performance at this one wavelength and angle of incidence, but not necessarily at other wavelengths and angles. There are two Epstein periods that will satisfy the requirement. There is a low-high-low index (LHL) period illustrated by the path $ABCZ$ and a HLH period as in $ADEZ$. The Epstein form uses 1st and 3rd layers that are the same thickness and a 2nd layer to suit. Generally there are only the 2 solutions that meet the symmetric 3-layer criterion and the relative thicknesses of the layers depend on the index to be approximated. Herrmann[10] described the nonsymmetric extension of the 3-layer simulation where the approximation is further improved for an extended wavelength region. Note also that there is a 2-layer nonsymmetric solution of HL in path AFZ. The first use of this design is attributed to Rock.[11] We will illustrate that the closer the admittance path of the approximation approaches the path of the layer being approximated, the more nearly the same as the approximated layer will be the performance of the design at all wavelengths and angles. This was also alluded to by Herrmann.[10]

III. Epstein Period Performance

The performance of the 2 Epstein periods used in the typical 3-layer QHQ broad band AR is shown in Figs. 4, 5, and 6. The solid curves are the design with the 1.65 index layer, the dotted curves are the LHL Epstein period, and the dashed curves are the HLH

The author is with Opto Mechanik, Inc., P.O. Box 361907, Melbourne, Florida 32935.

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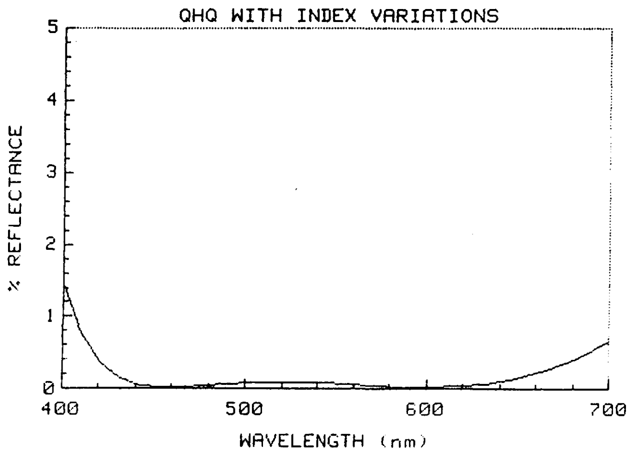

Fig. 1. Reflection of a broadband visible antireflection coating of QHQ design with indices 1.65, 2.1, and 1.38 on a 1.52 index substrate.

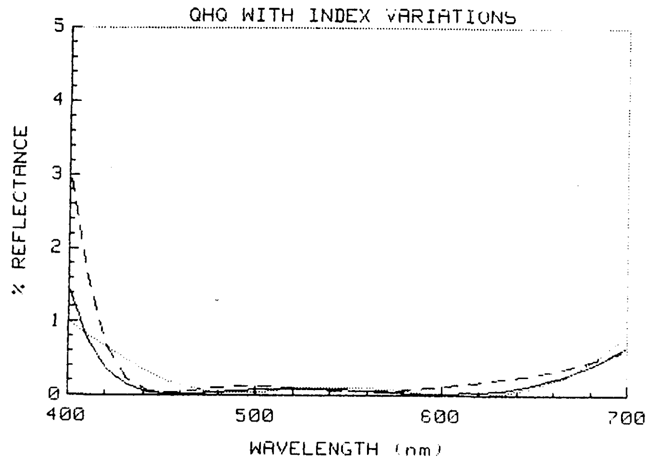

Fig. 4. Normal incidence reflectance of QHQ AR coating as in Fig. 1 with 1.65 index versions shown as solid line, Epstein LHL in place of 1.65 as ········, and HLH as – – – – – – – – – –.

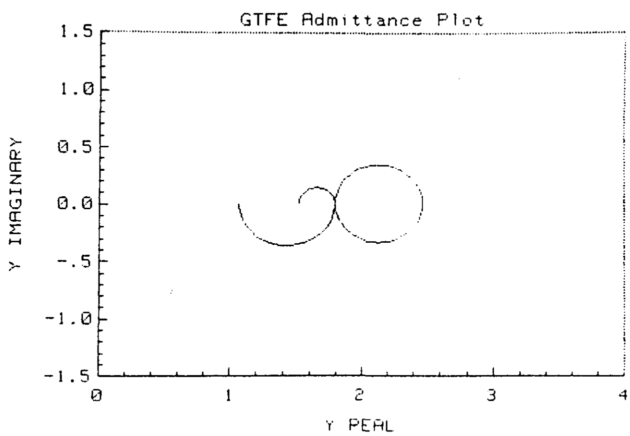

Fig. 2. Admittance diagram of the 3-layer coating in Fig. 1 at 524 nm and normal incidence.

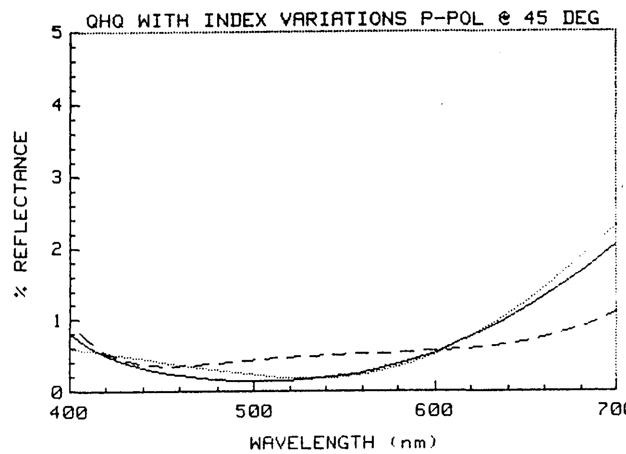

Fig. 5. 45° angle of incidence, P polarization reflectance as in Fig. 4.

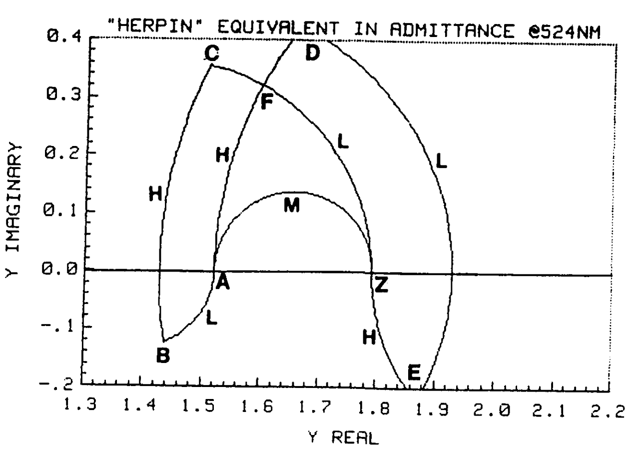

Fig. 3. Admittance of a QWOT of index 1.65 (curve M from A to Z), Epstein LHL period from $ABCZ$, Epstein HLH period from $ADEZ$, and the 2-layer Herpin equivalent HL from AFZ.

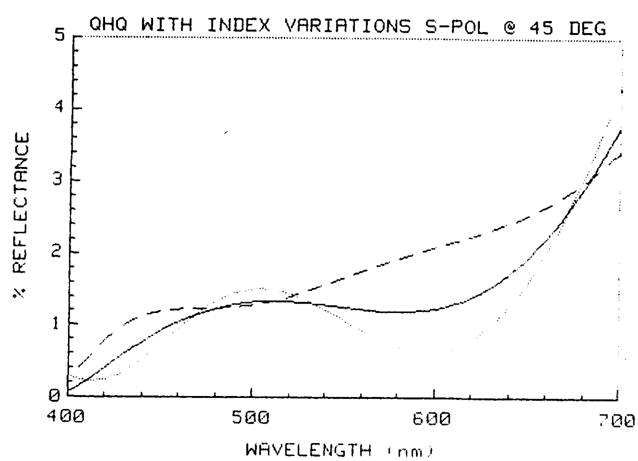

Fig. 6. 45° angle of incidence, S polarization reflectance as in Fig. 4.

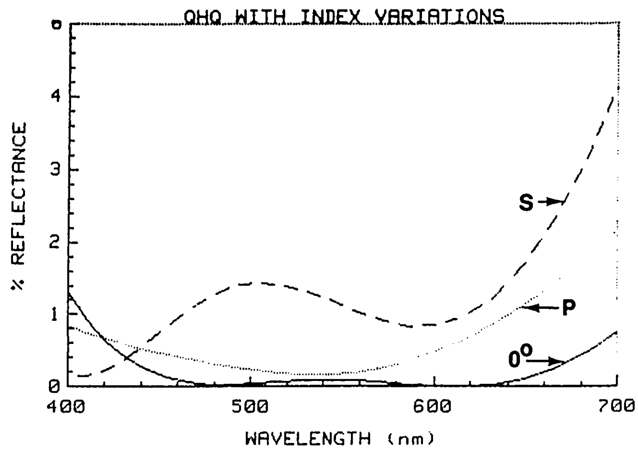

Fig. 7.
Reflection of 2-layer Herpin equivalent of 1.65 layer as in *AFZ* of Fig. 3.

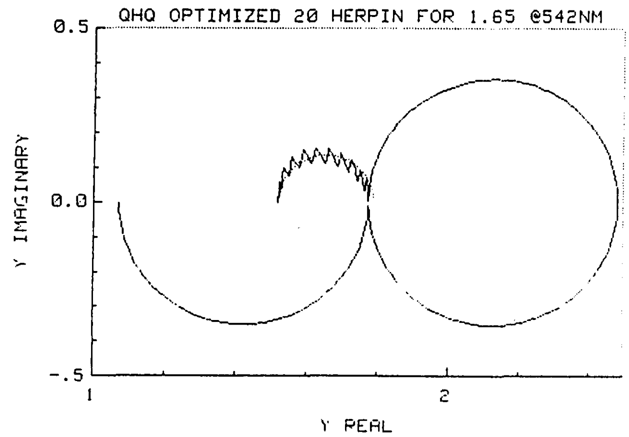

Fig. 8. Admittance diagram at 524 nm and normal incidence of the 3-layer AR coating with the 1.65 index layer replaced by the optimized 20 layer Herpin equivalent.

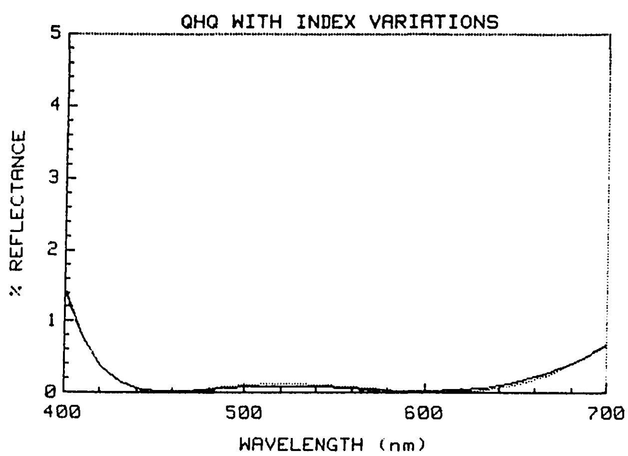

Fig. 9. Normal incidence reflection of the design in Fig. 8 ··········, solid line is the design of Figs. 1 and 2.

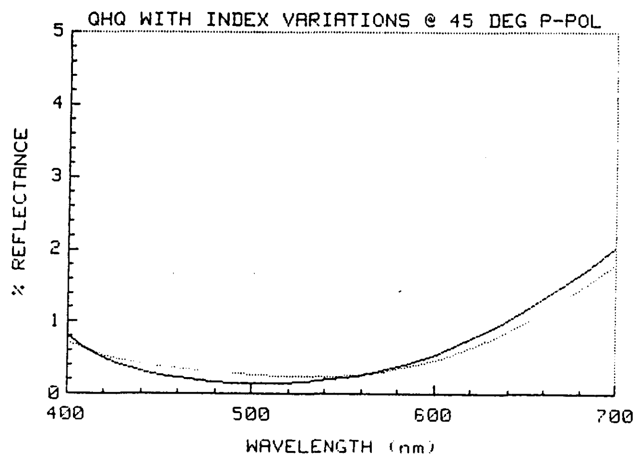

Fig. 10.
45° angle of incidence, *P* polarization as in Fig. 9.

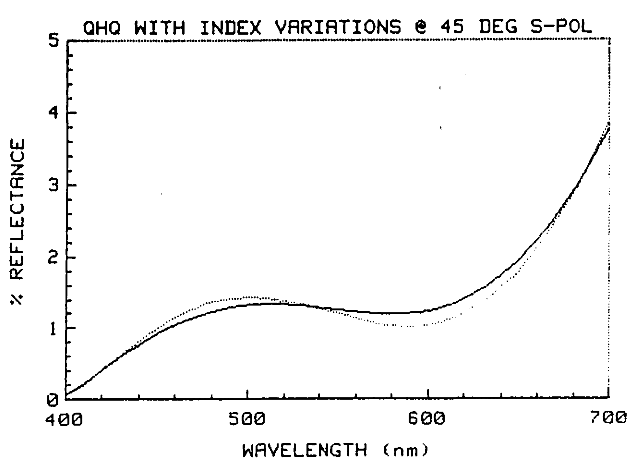

Fig. 11. 45° angle of incidence, *S* polarization as in Fig. 10.

Epstein period. Although the approximations are possibly practical solutions, they do illustrate the principles and the differences between the approximations. Note that the LHL curves are in all cases better than the HLH in replicating the 1.65 results. This can be inferred from Fig. 3 where the LHL path lies closer to the M path than the HLH. The S polarization at 45° angle of incidence shows the most dramatic differences of those illustrated.

IV. Non-Herpin-Epstein Approximations

Figure 7 shows the performance of the 2-layer approximation shown in Fig. 3 as *AFZ*. This is in fact closer to the 1.65 design than the symmetric solutions in this case. This may be inferred from Fig. 3 in that *AFZ* is everywhere closer to M than either the HLH or LHL approximations. Most of the four-layer/two-material ARs for the visible spectrum probably are closely related to this solution by design or accident. We automatically optimized a design with the LHL Epstein period plus a half wave of H followed by a QWOT of L where only the layers of the Epstein period were allowed to vary (layers 1, 2, & 3). The first layer was reduced to essentially zero thickness and the result was in fact the *AFZ* seen in Figs. 3 and 7.

These observations point intuitively to the possibility that a combination of many short layers which trace an admittance path very close to the path of the approximated layer should give a performance in all ways the same as the approximated layer. We took the optical thicknesses of the H and L layers of the two layer solution and divided them each into 10 equal parts which were alternated to make a 20-layer approximation of the 1.65 layer in a fashion after Southwell.[12] This produced a superior result to any of the approximations above, but we then let the automatic optimization vary the thickness of each of the 20 layers for further improvement. The resulting admittance diagram of the design is shown in Fig. 8. It will be seen that the result is an admittance path which follows the 1.65 admittance path very closely. Figs. 9, 10, and 11 compare the performance of the 20-layer approximation with the design using the 1.65 index. It can be seen that the differences are almost imperceptible and could be reduced to imperceptibility by further refinement.

V. Conclusions

We conclude and have illustrated that the Herpin equivalent index and Epstein period approximations are exact at one wavelength and angle of incidence, but give different results at other angles and wavelengths. There are, in some cases, 2-layer approximations that are better than the symmetric equivalent. Any intermediate index can be theoretically replaced in detail performance by a combination of many layers of bounding indices to any desired degree of accuracy if enough of the proper finely divided layers are used to follow closely the desired admittance path.

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