Maximizing production yield and performance in optical instruments through effective design and tolerancing

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ABSTRACT

The focus of this paper is concerned with the practical aspects of designing optical instruments which are intended for production in large or even small quantities. Certain aspects of performance are required of an instrument which is its reason to exist. It is usually expedient, plus fiscally and ecologically sound, to strive for an overall efficient use of resources in the life cycle of the instrument (i.e., keep the net cost down). This includes the processes from concept through design, prototypes, production, use, and disposal. The design and tolerancing aspects of the process have a major effect on the life cycle cost and efficiency of the system and that is the principal subject of this paper.

We discuss what makes up the cost of a lens and the effects of tolerances and other factors on that cost. This results in a new lens cost estimation formula. We describe the interactions of lenses and lens cells from the tolerance viewpoint. We then explain the principles whereby the system tolerances can be determined which will give the minimum cost system which meets the performance requirements. We conclude with an example from real life of the preliminary application of the principles.

1. INTRODUCTION

The assignment of tolerances to the various dimensions and parameters of an optical system is a CRITICAL element in determining the resulting performance and cost of the system. This has historically been complex and ill-defined and therefore a distasteful aspect which the designer has had to deal with in one way or another. Much of the tolerancing of systems in practice has been done by art and experience more than by scientific calculation. Here, we attempt to make the ENGINEERING principles as simple and clear as possible so that they may be applied in a straightforward manner. We use the term "engineering" to imply that practical approximations based on empirical data are used to reduce the problem to terms that can be handled in the real world. That is to further imply that we are dealing here with the application of optical design to produce a functional set of hardware as opposed to the theory and/or science of lens design as an exact and abstract study. In the production of an optical
system, random errors in parameters occur. These cause the results to be statistically predictable, but not exactly calculable. Therefore, the use of reasonable engineering approximations is appropriate and justifiable.

It is sometimes possible to tolerance a system such that each of the components is fabricated to an accuracy which will insure that the instruments will be adequately precise and aligned to give the required performance by simple assembly with no alignment or adjustment. This may be the case in certain diamond turned optical assemblies. The other extreme is where almost every parameter of a system can be adjusted to bring the system to the desired performance. However, neither of these approaches is usually the least cost way to meet the performance requirements. We discuss philosophical principles and practical ways of approaching the least cost solutions and give an illustration of the application of the techniques.

This paper is the forth of a series (1,2,3) which started in 1982 where the authors investigated how to achieve the least-cost tolerancing of an optical system. Since that time, Wiese (4) has compiled a very useful collection of papers on tolerancing in its many aspects. That volume includes a paper by Plummer (5) which played a role in our earlier work and a paper by Adams (6) which judiciously utilized some of our findings. Fischer (7) did a more recent survey based on Plummer's work which we compare with some of our updated cost versus tolerance data below. Parks (8,9) and Smith (10) have papers in Wiese's collection that have many practical and helpful discussions on the subject. The works referenced form a good general background for this paper, but we will reiterate the salient points below for the convenience of the reader. We will also cite other specific references as they apply below.

It is the authors' experience and opinion that there has been a great deal of waste resources in the past due to poor tolerancing "art". A rigorous and all encompassing treatment of all but the simplest system can be VERY complex. It is our aim to move the practice of tolerancing from the art stage to the engineering stage with as much simplification as is reasonably justifiable. Warren Smith (10) has done a great deal to move the status of the practice in this direction, we are attempting to move another step along the way by adding the real influence of cost into the tolerancing process.

The optical and mechanical designers of instruments have by far the greatest influence on the ultimate cost and performance of an instrument. All others, including the manufacturing operations, cannot have as much power as the designers to change the potential satisfaction of the user and profitability of the producer. Figure 1 illustrates this graphically. Figure 2 outlines the detail design process and how the subjects of this paper interplay in the process.
FIGURE 1. OVERALL PROCESS TO DEVELOP AND PRODUCE A NEW OPTICAL PRODUCT

FIGURE 2. DETAIL DESIGN AND ANALYSIS PROCESS OF AN OPTICAL PRODUCT
2. BASE COSTS

We will first review the concepts of base costs from our previous work and then add the results of new work on the estimation of the base costs directly from the data on a drawing and/or the specifications of a given component.

Let us take the example of fabricating a single lens. If someone asked that we make a biconvex lens of glass, we would typically have to go through the following steps:

- generate (or mill) a radius on the two sides
- mount the lens on a spindle
- grind and polish the first side of the lens
- remount the other side of the lens on a spindle
- grind and polish the second side
- edge the lens

There are obviously a few other small steps such as obtaining materials and grinding and polishing tools, dismounting, cleaning, etc. We have said nothing to this point about further specifications such as diameter, radii, thickness, and tolerances. Without these there is still a cost in time and materials and equipment necessary to make the biconvex glass lens. This is what we call the base cost. As we get more specific about the lens and add more restrictive tolerances on the parameters, more care and time and equipment will probably be required to make the lens to the new specifications. Therefore the cost will increase with increasingly stringent requirements/tolerances/specifications.

Our previous work (1,2,3) went into some detail on the relationships of the increase above the base cost with changing tolerance values. However, neither we nor Plummer(5) quantified those base costs. We will quantify the base costs in this work and thereby add significantly to the usefulness of the technique and somewhat to the accuracy of the results. What was not made clear in the earlier work was that the base costs to be used in a given cost versus tolerance case are not the same for grinding and polishing as they are for centering and edging. As we shall show below, the centering and edging operation is not influenced by grinding and polishing costs or tolerances and vice versa. Not incorporating this concept can lead to some error in the application of our earlier work and Adams'(6) extension of it. There is a different base cost to which the increase with tolerance is to be applied for the two classes of operation. Figures 3 and 4 show the tolerance of lens diameter and eccentricity costs as a percentage of base cost for the centering and edging operations. This applies to the centering and edging base cost (CE). Figures 5 through 10 show the tolerance costs as a percentage of the base grinding and polishing costs (GP) for tolerances of radius of curvature, irregularity, diameter to thickness ratio, center thickness, scratch and dig, and the glass stain.
**% BASE CENTERING-EDGING COST VS LENS DIAMETER TOLERANCE**

\[ \text{KD} = 12.5 \times 10^{-6} \text{ IF } \Delta d \text{ IN INCHES} \]
\[ \text{KD} = 0.316 \text{ IF } \Delta d \text{ IN MICROMETERS} \]

\[ \text{COST} = \text{BASE} \times \left(1 + \frac{10 \times \text{KD}}{\Delta d}\right) \]

- **\( \bullet \)** = PLUMBER'S TABLE 1 DATA
- **\( \Delta \)** = AUTHOR'S EXPERIMENTAL DATA
- **\( f \)** = FISCHER'S AVERAGE

**FIGURE 3: LENS DIAMETER TOLERANCE (PLOTTED ON RECIPROCAL SCALE)**

**% BASE CENTERING-EDGING COST VS ECCENTRICITY TOLERANCE**

\[ \text{KW} = 1 \text{ IF } \Delta A \text{ IN MILLIRADIANS, WEDGE} \]
\[ \text{KW} = (n-1) \times 3.44 \text{ IF } \Delta a \text{ IN MINUTES OF ARC, DEVIATION} \]

\[ \text{COST} = \text{BASE} \times \left(1 + \frac{145 \times \text{KW}}{\Delta A}\right) \]

- **\( \bullet \)** = PLUMBER'S TABLE 1 DATA
- **\( f \)** = FISCHER'S AVERAGE (ASSUMING A 2"DIA PART)

**FIGURE 4: ECCENTRICITY TOLERANCE (PLOTTED ON RECIPROCAL SCALE)**
characteristics. It was also interesting to us to recognize that the milling or generating costs are not particularly affected by tolerances with today's equipment and they are not part of the base which the tolerance factors multiply. The milling costs are therefore part of the base, but only a function of material and dimensions, not tolerances.

Before we discuss the cost versus tolerances in detail in the next section, we will show the development of the base cost formula. The present view and cost estimating scheme is the authors best effort to date resulting from their experience and discussions drawing on the work and experience of Stephen Cupka, Manager of Estimating and Reinhard Seipp, Assistant Manager of Optical Manufacturing at Opto Mechanik, Inc., most of these persons also have experience from one or more other shops than OMI to draw upon.

Let us call the total base cost to make a lens $MT$, which is in either time or money which differ only by some multiplicative factor. If we call the milling or generating cost $MG$, we can represent the total base costs as the sum of milling, grind and polish, and centering and edging costs as given in Eqn. 1.

$$MT = MG + GP + CE$$

(1)

### 2.1 Milling/generating costs

We find a reasonable fit with experience in milling to be given in Eqn. 2 where $LM$ is the number per lot to be milled and $d$ is the diameter in inches. The milling of both sides of the lens is included here.

$$MG = 4 + 90/LM + 0.1d^2$$

(2)

This implies that there is some base cost to mill the optic plus some setup cost divided by the number of parts to be run from that setup plus a factor due to lens size. Since the material to be removed from a molded blank is usually about the same thickness independent of blank diameter, the generating time is only a function of a blank's area ($d^2$).

### 2.2 Centering and edging costs

We will deal with $CE$ before $GP$ because it is simpler. The cost is a function of the number in the lot $LC$ to be centered on one setup, the diameter $d$, the number of chamfers $C$, and the number of flats $F$ (planes perpendicular to the lens axis). Equation 3 represents our collective best estimate.

$$CE = (2 + d + C + F)/3 + (30 + 10C + 15F)/LC$$

(3)

This accounts for a setup cost for the diameter, chamfers, and flats plus the edging of each lens.
2.3 Grinding and polishing costs

The grinding and polishing cost is a very strong function of the number of lenses which can be ground and/or polished at one time on a block. If the radius is short the number NS which can be blocked for that side is determined by the radius R and the lens diameter d. If the radius is long, the number which can be blocked is determined by the maximum block diameter G and the lens diameter d. The precise calculation of this number can be done when flats, chamfers, and center thicknesses are properly accounted for. That subject might be worthy of a paper on its own. For simplicity and practicality, we will use a conservative approximation without showing its derivation here. If the R/d is less than .87, only a single lens can be polished at one time, and at least 3 per block can be done if the ratio is greater. If the radius is short the number per block NS will be given by integer value in Eqn. 4.

\[ NS = \text{INT}(2.9 \times (R/d)^2) \]  

(4)

Here R is the radius and d is the diameter of the side in question, and N1 would be NS for side 1 and N2 would be NS for side 2.

If the radius of the side is long, the number per block will depend on the diameter G of the largest block which can be used as shown on Eqn. 5.

\[ NS = \text{INT}(0.64 \times (G/d)^2) \]  

(5)

Whichever NS is smaller for a given side (for R or G) will be the number which can be ground and polished per block for that side. Our experience is that less than the maximum number will often be used when the radius is long. This is not of great cost consequence, however, because the change in cost with a few parts more or less when NS is large is seen in Eqn. 6 to be small relative to other costs.

Our collective experience with the grind and polish costs GP per lens and per side in a block of NS lenses is given in Eqn. 6.

\[ GP = 7 + 14/NS \]  

(6)

It is also usually appropriate to consider a yield related factor Y due to scrapage, etc. This is the factor times the number of lenses to be delivered which must be started to give the yield; it is actually inverse yield then. When this is applied and both sides are considered, the total GP base costs are given in Eqn. 7 where N1 and N2 are the numbers of lenses per block for side 1 and side 2.

\[ GP = Y \times (14 + 14 \times (1/N1 + 1/N2)) \]  

(7)

We now have all of the components of the base cost (in relative units) for a lens based on a collective empirical history and in a fairly workable form. We only need to
apply the effects of tolerances and other influences such as material properties and diameter to thickness ratio to these base costs in the next section and we can then predict the cost to produce a given lens to its specifications. The derivative of that cost with a change in any tolerance or parameter will be used to determine the distribution of tolerances which will minimize system cost and insure the desired yield.

3. THE EFFECT OF TOLERANCES AND OTHER FACTORS ON COSTS

We have generally updated and included here the cost versus tolerance data presented in our previous work (3) in Figs. 3 through 10. The dots were Plummer's work (5), the triangles were our previous experimental and or estimated values, the c's are factors from Cupka's earlier estimates, and the f's are the averages from Fischer's survey (7). The lines drawn are the functions which we currently choose to use as the best estimate from experience and the functional equations are indicated on each figure. On the figures where appropriate, we have included information from Smith(10). He indicated tolerances that he thought we "low cost" which we have plotted as "$". We show his "commercial" as "$\$", "precision as "$\$$", and "extra precise" is shown as "$\$$\$$. These are in general accord with other data. For a more extensive discussion of the previous work, see the references.

Figure 3 shows the cost effect of lens diameter tolerances which is not a strong factor up to the limit of the capabilities of the edging process. The various authors' data are in good agreement. We have shown the tolerance scale in inches, and micrometers on this and some of the other figures for the readers convenience.

Figure 4 shows the cost effect of the lens centering tolerances. The centering tolerance is sometimes expressed as light deviation, wedge, or total indicator runout (TIR) at the edge of a lens or window. These different versions of the requirements can be reconciled in the following way. The wedge angle in radians is the same as the TIR divided by the diameter being measured. The deviation depends on the wedge times the refractive index minus one and conversion to arcminutes as needed. The KW-factors in the figure account for this. Plummer's and Fischer's data are in good accord and are well represented by the approximation which we use.

Figure 5 deals indirectly with the cost of radius of curvature tolerances. The work of Thorburn (11) reported in Wiese's collection made it clear that we should refine our approach to radius of curvature. We had previously just used the percentage of the radius as a measure of stringency of the radius tolerance. But it becomes apparent after a little analysis that the delta in sagitta over a surface is much more meaningful. This is what a spherometer or an interferometer can measure. One can derive that the change in sagitta with a change of radius is approximated by Eqn. 8.

\[ \frac{dZ}{dR} = 0.125 (d/R)^2 \tag{8} \]
% BASE GRIND & POLISH COST
VS
RADIUS OF CURVATURE TOLERANCE
EXPRESSED IN SAGITTA, ΔZ

COST = BASE GP (1 + KZ / ΔZ)

KZ = 12.5 x 10^-4 IF ΔZ IN INCHES
KZ = 1 IF ΔZ IN FRINGES

CONVERSION FOR ΔZ TO ΔR IN INCHES:
R = ΔZ x 8(R/d)^2

CONVERSION FOR ΔR TO ΔZ IN INCHES:
Z = ΔR x 125(d/R)^2

FIGURE 5: RADIUS OF CURVATURE TOLERANCE EXPRESSED IN SAGITTA ΔZ (PLOTTED ON RECIPROCAL SCALE)

% BASE GRIND & POLISH COST
VS
LENS FIGURE IRREGULARITY TOLERANCE

KI = 12.5 x 10^-4 IF ΔI IN INCHES
KI = 1 IF ΔI IN FRINGES

COST = BASE GP (1 + 25KI / ΔI)

FIGURE 6: IRREGULARITY TOLERANCE ON LENS FIGURE (PLOTTED ON A RECIPROCAL SCALE)
This is what we now use as seen in Fig. 5. The graph is plotted as cost versus the reciprocal of the delta sagitta $dZ$ which is a function of delta radius $dR$ and $R$ and $d$. The delta $R$ and $R$ alone are not enough to estimate the real cost of the tolerance, the surface diameter must also be taken into account. It might be more appropriate to refer to the sagitta tolerance of a surface than the radius tolerance, but shops are used to seeing the radius tolerance spelled out on a drawing. Equations 9 and 10 can be used to convert either way between $dR$ and $dZ$ (typically in inches).

$$dR = dZ * 8 \frac{R}{d} \times 2$$

$$dZ = dR * 0.125 \frac{R}{d} \times 2$$

The cost function chosen is based on Thorburn's (11) statement that a tight tolerance on sagitta is .000005 inch and .0001 inch is loose.

Figure 6 shows the impact of surface figure irregularity tolerances. We described the earlier experimental data in Ref. 2. We have added the points from Fischer and Cupka and a slight reformulation and labeling. The data from the various sources seems sufficiently consistent and is reasonably represented by the chosen function.

Figure 7 shows the effects of the diameter to thickness ratio of a lens. This has to do with the flexibility of the part when trying to hold a good surface figure and sometimes the temperature effects when working thin negative lenses. Our simplistic treatments of this to date are probably not as adequate as we would like. This might be worthy of a paper by itself. Fischer (7) reported a much less severe effect than our previous work (3). In our work we estimated that the effect was inversely proportional to the flexibility of a disk which goes as the cube of the diameter to thickness ratio $d/T$. We will now accept (conditionally) Fischer's data collection as possibly more representative and we have fit our function to a compromise between our old data and that of Fischer where both are seen in Fig. 7. There is little doubt that plano-plano windows can be worked very thin by contacting them to rigid plates which lend the effect of their $d/T$ ratio. In the case of lenses, this is not the practice however. Structural shape and thermal effects, etc. need to be studied in more detail. We will use this function until we or someone else can refine it further.

Figure 8 deals with the cost of center thickness tolerances. A major effect here is that once a thickness goes under the tolerance limit it is lost and must be replaced. We believe that this is what is reflected in the radical change in cost in Plummer's data as a .0005 inch tolerance is approached. At some point, a given tolerance would not allow a scratch or pit to be ground or polished out and the part would be lost due the combination of the thickness and scratch and dig specifications. Plummer's and Fischer's data are in reasonable agreement, and Cupka only
Figure 7: Cost effect of difficulty in obtaining good figure irregularity due to flexures from diameter to thickness ratio of part (cubic scale).

Figure 8: Lens center thickness tolerance (plotted on a reciprocal scale).
divided the costs into above and below a .002 inch thickness tolerance.

Figure 9 shows the effects of scratch and dig tolerances. The various data cluster reasonably near the very simple function that we choose to employ.

Figure 10 shows the simple function of cost versus glass stain code that we fit to our previous data and Fischer's. We might comment that we feel that even the 5 code glasses can be worked on a fixed price basis now, but at a significant added cost.

The "polishability" factor should also be applied as a cost impact. Each shop has its own experience that pyrex takes more time to grind and polish than BK7 and fused silica takes longer than pyrex, etc. In an attempt to include this factor, we have collected the estimated time factor for a variety of materials as compared to BK7. Some suggest that germanium has the same polishing time as BK7, and others think it takes significantly longer. This will, of course, vary from shop to shop and the procedures used. The author believes that typically germanium lenses will polish to spec in about the same time as BK7, but that the typical polish spec may be 160/100 for Ge and 80/50 for BK7. Therefore, the Ge will take longer to meet all of the same specs as a piece of BK7. The numbers in Table 1. reflect the author's best estimate based on a variety of inputs, but each shop needs to examine which factors to use in their own case. The grind and polish time will then be multiplied by this material polishability factor P.

TABLE 1.

Polishability Factor of Various Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK7</td>
<td>100%</td>
</tr>
<tr>
<td>SF56</td>
<td>120%</td>
</tr>
<tr>
<td>Pyrex</td>
<td>125%</td>
</tr>
<tr>
<td>Germanium</td>
<td>130%</td>
</tr>
<tr>
<td>Fused Silica</td>
<td>140%</td>
</tr>
<tr>
<td>Zerodur</td>
<td>150%</td>
</tr>
<tr>
<td>ZnS, ZnSe</td>
<td>160%</td>
</tr>
<tr>
<td>FK2, BaF2, Amtir</td>
<td>170%</td>
</tr>
<tr>
<td>LaKN9, LaFN21</td>
<td>200%</td>
</tr>
<tr>
<td>Electroless Ni</td>
<td>250%</td>
</tr>
<tr>
<td>CaF2, LiF</td>
<td>275%</td>
</tr>
<tr>
<td>MgF2, Si</td>
<td>300%</td>
</tr>
<tr>
<td>Electrolytic Ni</td>
<td>350%</td>
</tr>
<tr>
<td>Ruby</td>
<td>700%</td>
</tr>
<tr>
<td>Sapphire</td>
<td>800%</td>
</tr>
</tbody>
</table>

Once the lenses are fabricated, they are typically mounted in metal cells. We have to coordinate the tolerancing of the metal and the glass to get the desired results. Similar cost versus tolerance curves can be developed.
% BASE GRIND & POLISH COST
VS BEAUTY DEFECTS

COST = BASE GP \left( 1 + \frac{10}{S} + \frac{5}{D} \right)

\( \Delta \) = AUTHOR'S EXPERIMENTAL DATA
S = SCRATCH#
\( \ell \) = FISCHER'S AVERAGE
D = DIG#
\( \gamma \) = CUPKA'S FACTORS

FIGURE 9: EFFECT OF SURFACE FINISH REQUIREMENTS

% BASE GRIND & POLISH COST
VS WORST GLASS STAIN CHARACTERISTICS

COST = BASE GP \left( 1 + 0.01 \times S \right)^3

\( \Delta \) = AUTHOR'S PREVIOUS POINTS
\( \ell \) = FISCHER'S AVERAGE

FIGURE 10: EFFECT OF GLASS STAIN CHARACTERISTICS
Figure 11 shows the cost versus lens cell diameter tolerance for both manual and automatic machining. The same curve applies to the length tolerances between bores as in spacers. Once an automatic is setup, the parts repeat within the capability of the machine with little change in the cost versus tolerance. The manual operations are more and more labor intensive as the tolerances increase.

Figure 12 shows the cost versus tolerances of bore concentricity and length run-out such as tilt in a spacer. There is no difference from manual to automatic here. However, the big difference here is whether both bores are cut without removing the part from its holder (chuck). If so, then the concentricity will be limited only by the accuracy of the machine. If the part must be rechucked for the second bore, much more time is consumed to hold a tight tolerance in the rechucking or mandrill type operation. There is a strong motivation to design for single chucking as much as practical.

We will use the results of Figs. 11 and 12 when we come to distributing the tolerances in the instrument design. The base cost for the machining operation will be needed to multiply times the metal tolerance factors in this process. For simplicity we will ignore setup costs and use a base machine fabrication cost of 6 units per bore or spacer cut in the same denomination as those used for the lenses above. For an automatic, we will use 4 units. This aspect could be made much more complex and refined, but this approximation is adequate to allow us to properly distribute tolerances in the later sections, but not estimate total machining costs.

These cost versus tolerance and cost versus other parameter functions are the major ones influencing the optical component fabrication cost. Now that we have them reasonably characterized, we can apply them to the base costs given in Sec. 2 to estimate the total component cost as specified. We can also find (to a sufficient approximation) the change in total component cost with a change in any of the parameters or tolerances. This, coupled with the sensitivity of the system performance characteristics to each of the parameter and tolerance changes, will then allow us to distribute the tolerances in such a way as to minimize the system cost while maintaining the required successful product yield. This is the major goal toward which we now strive.

4. TOTAL LENS COST ESTIMATION

When we combine the base costs given in Eqns. 2, 3, and 7 with the tolerance and other factors on Figs. 3 through 10 and Table 1, we get the total lens cost estimation given in Eqn. 11. This obviously cannot be scientifically rigorous, but is only a practical estimate for engineering or business purposes. The detail factors will vary from shop to shop and time to time. It is the author's belief that this sort of an estimation formula will be more accurate and consistent than most estimators or shop cost records now.
% BASE METAL FABRICATION COST

* vs DIAMETER TOLERANCE ($\Delta dM$),
  ALSO APPLICABLE TO LENGTH TOLERANCE ($\Delta L$)

MANUAL MACHINING, COST = BASE $\times (1 + \frac{20KM}{\Delta dM})$

$KM = 12.5 \times 10^{-6}$ IF $\Delta$ IN INCHES
$KM = .316$ IF $\Delta$ IN MICROMETERS

AUTOMATIC MACHINING, COST = BASE $\times (1 + \frac{2KM}{\Delta dM})$

$\Delta dM$ $\Delta L$

FIGURE 11: LENS BORE DIAMETER OR LENGTH TOLERANCES (PLOTTED ON THE RECIPROCAL SCALE)

% BASE METAL FABRICATION COST

* vs CONCENTRICITY TOLERANCE ($\Delta CE$)
  ALSO APPLICABLE TO TILT OR LENGTH RUN-OUT ($\Delta LE$)

MANUAL MACHINING, COST = BASE $\times (1 + \frac{80KM}{\Delta CE})$

IF PART MUST BE RE-CHUCKED FOR 2nd SURFACE

AUTOMATIC MACHINING, COST = BASE $\times (1 + \frac{4KM}{\Delta CE})$

$KM = 12.5 \times 10^{-6}$ IF $\Delta$ IN INCHES
$KM = .316$ IF $\Delta$ IN MICROMETERS

FIGURE 12: LENS CELL BORE CONCENTRICITY OR LENGTH RUN-OUT TOLERANCE
(PLOTTED ON RECIPROCAL SCALE)
TOTAL LENS COST = MT =

(GENERATING) + MG

(PART SETUP) + Y*14

(SIDE 1, G&P) + (P*Y*14/N1)((1+25*KI/\Delta I1)*(1+(R1/d)^2)*B*KZ/\Delta R1+.0003*(d/T)^3)
+40*KT/\Delta T*(1+10/S1+S5/D1)*(1+.25*KI/AI1)*(1+(R1/d)^2*8*KZ/AR1+.0003*(d/T)^3)
+40*KT/\Delta T*(1+10/S2+S5/D2)*(1+.25*KI/AI2)*(1+(R2/d)^2*8*KZ/AR2+.0003*(d/T)^3)

(SIDE 1, G&P) + (P*Y*14/N2)((1+25*KI/\Delta I2)*(1+(R2/d)^2)*B*KZ/\Delta R2+.0003*(d/T)^3)
+40*KT/\Delta T*(1+10/S2+S5/D2)*(1+.25*KI/AI2)*(1+(R2/d)^2*8*KZ/AR2+.0003*(d/T)^3)

(CENTERING) + CE*(1+10*KD/\Delta d+K*W/\Delta A)

Although Eqn. 11 is extensive, it is not particularly complex. Table 2 summarizes the parameters of the equation.

**TABLE 2**

<table>
<thead>
<tr>
<th>MT</th>
<th>Total lens cost estimate per piece (in relative units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MG</td>
<td>Milling/generating cost from Eqn. 2</td>
</tr>
<tr>
<td>LM</td>
<td>Number of parts milled in one lot setup</td>
</tr>
<tr>
<td>Y</td>
<td>Yield factor of parts started/parts acceptable</td>
</tr>
<tr>
<td>P</td>
<td>Polishability factor from Table 1</td>
</tr>
<tr>
<td>N1</td>
<td>Number of parts/block for R1 side from Eqns. 4 or 5</td>
</tr>
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The generating and certain part setup costs are virtually independent of the tolerances of the part. The centering and edging operation and tolerances are virtually independent of the grinding and polishing operations and tolerances. The operations and tolerances of the two radii of the lens are in most senses independent. This is all reflected in Eqn. 11. There has been some logic applied to whether the tolerance factors are added or multiplied and to what they are applied. The irregularity cost factor multiplies times the base cost of grinding and polishing each side. That irregularity cost is further multiplied by the radius tolerance cost factor.
plus the flexure \((d/T)\) difficulty factor which make the figure irregularity harder to achieve. The decision to add the \(d/T\) influence to the radius tolerance influence was made on the basis that they do not significantly effect each other, but they both effect the cost of meeting the irregularity specifications. The stain class interacts with and effects the scratch and dig requirements and they both increase the difficulty of holding the thickness tolerance, but they do not effect the irregularity degree of difficulty. The edging operation is affected by the diameter tolerance and the wedge or deviation tolerance. We will discuss the unique interaction of the diameter and centering tolerances in the next section.

This formula then reduces the estimation of the fabrication cost of most lenses to a clerical task of entering the parameters from the lens drawing into a simple computer program. This could also, in principle, be worked into a CAD program to allow the designer to see immediately the cost impact of the design and tolerances. The key factor for tolerancing, however, is that we can find the partial derivatives of Eqn. 11 with respect to each of the reciprocal tolerances for use in finding the minimum cost tolerance distribution for a system as discussed in Sec. 6.

5. INTERACTIONS OF LENSES AND MOUNTS

A lens system typically consists of lenses in metal mounts. The mounts are bores that closely fit the lens diameters with "lands" or rings of contact between one or both of the radii and a metal locating surface in the direction of the optical axis. The relationship of one lens to another will be determined by the spacing dimensions and tolerances of the mount and the concentricity and tilts between the locating surfaces. The mounting metal (or other material) must be tolerated to be compatible with the glass tolerances. In this section we will address how we deal with this task.

There are several interesting references (11,12,13) dealing with tilts, decentrations, and rolls (by whatever names) but none seem to have addressed them in an analytic form appropriate to our cost minimization goals.

5.1 Lens Centering

Figure 13 shows a lens in a cell bore where only centering factors are considered, not tilts. It is easy to evaluate the effect or sensitivity of decentering a lens from the intended optical axis in most lens design software. This decentering in a system is the sum of several factors. The decentering of the optical axis of the lens with respect to the outside diameter of the lens is what the optical shop works on. The centering of the mounting bore with respect to the ideal axis is what the machine shop works on. There needs to be some clearance for assembly to insert the lens into the cell, we will call this \(f\). The tolerances of the lens diameter \(d\) and the bore diameter \(dM\), delta \(d\), and delta \(dM\) give rise to more potential clearance. These clearances will allow the lens to move to extreme
FIGURE 13: DECENTERING FACTORS OF A LENS IN A CELL

FIGURE 14: TILT AND ROLL FACTORS OF A LENS IN A CELL
positions in the cell, such as fall to the bottom, and cause more
decentering. Equation 12 expresses the possible total decentering \( td \) as a
function of the lens decentering \( dc \) and the factors mentioned above.

\[
td = dc + \left( f + \delta d + \delta dM \right)/2
\]  
(12)

The costs versus tolerances have been defined and quantified in Sec. 3. We
can see that the least cost distribution of tolerances for a lens in a bore
with respect to total decentering will dictate a certain ratio between the
\( dc \), \( \delta d \), and \( \delta dM \). We will then reduce the tolerancing of the
"set" to a function of only \( \delta d \) since the \( dc \) and \( \delta dM \) can be taken
as the dependent variables.

The \( dc \) of the lens is measured by the wedge \( \Delta A \) in Fig. 13 (here we
work in milliradians) or in the arcminutes deviation it causes which we
relate by the factor \( KW \) in Fig. 4. We can show that \( dc \) can be expressed in
terms of \( \Delta A \) as in Eqn. 13. (Some RSS possibilities will be ignored
here.)

\[
dc = \left( \frac{R1 \cdot R2}{(R2-R1)} \right) / \left( 1000KW \right) \cdot \Delta A
\]  
(13)

\( \Delta A \) in terms of \( \delta d \) is found to be

\[
\Delta A = \sqrt{\frac{0.145KW}{10KD}} \cdot \delta d
\]  
(14)

If we call the base machining cost \( MF \), then \( \delta dM \) in terms of \( \delta d \) is
given by Eqn. 15.

\[
\delta dM = \sqrt{\frac{MF \cdot 20KM}{CE \cdot 10KD}} \cdot \delta d
\]  
(15)

The minimum fit clearance factor \( f \) has to be determined by the assembly
plans for the cell and whatever allowances are made for differential thermal
expansion. At nominal temperature, it will allow a shift in an otherwise
perfectly fitting cell of \( f/2 \). This decentering will have to come right off the
top of the total decentering budget for this lens+bore set leaving the
residual budget to be partitioned among \( \delta d, \delta dM, \) and \( \Delta A \). We express
the result in Eqn. 16.

\[
td-f/2 = \left( \frac{1}{2} + \sqrt{\frac{MF \cdot 20KM}{CE \cdot 10KD}} \right)
+ \sqrt{\frac{0.145}{(10KD \cdot KW)}} \cdot \text{ABS}\left(\frac{R1 \cdot R2}{((R2-R1) \cdot 1000)}\right) \cdot \delta d
\]  
(16)

This factor times the \( \delta d \) is to be used in the tolerance allocation
process with the decentering sensitivity to determine \( \delta d \). The \( \delta d \)
can then be used to assign \( \Delta A \) and \( \delta dM \) (which are dependent on
\( \delta d \)) in a secondary operation using Eqns. 14 and 15. The diameter of
the cell bore \( dM \) is also determined by this process as expressed in Eqn.
17.

\[
dM = d + f + \delta d + \delta dM
\]  
(17)
Although Eqn. 16 might not appear to simplify anything, it is a relatively straightforward application of the cost and geometric factors which allow us to properly spread the cost and tolerances in the lens and its cell.

5.2 Lens tilt and roll

An otherwise perfect lens might be tilted with respect to the system's ideal optical axis because the metal locating surface of the cell is tilted by an angle \( \Delta \alpha \) as shown in Fig. 14a. This can be simply dealt with using the cost versus tilt curve in Fig. 12 and the sensitivity of the system to tilt of the whole lens. This assumes that the tilt is not otherwise limited by such factors as a retainer on the other side of the lens or the fit of the cylindrical lens diameter into the cylindrical cell bore which prohibit that much tilt.

The perfect lens might also "roll" in an oversized bore as shown in Fig. 14b. This shows that the left hand surface tilts while the right is correctly located against the "perfect" cell. The lens will roll about the center of curvature of the right hand surface \( R_2 \). The left hand surface, which we have shown plano for clarity, will tilt through an angle of \( \Delta \alpha R \) which is approximately \( \frac{V}{R_2} \) radians. We can show that:

\[
V = \left( f + \Delta d + \Delta d_M \right) / 2. \tag{18}
\]

We know \( \Delta d_M \) in terms of \( \Delta d \) from Eqn. 15. This allows us to express \( \Delta \alpha R_1 \) of the left surface \( R_1 \) as a function of \( \Delta d \) in Eqn. 19.

\[
\Delta \alpha R_1 = \left( f + (1 + \text{SQR}(\text{MF}^2 \text{KM}/(\text{CE}^2 \text{KD}))) / \text{ABS}(2^2 \text{R}^2) \right) \times \Delta d \tag{19}
\]

This and the system performance sensitivity to a tilt of \( R_1 \) will allow us to allocate the tolerance budget for a tilt of \( R_1 \). However, note that controlling this requires the control of \( \Delta d \) which is already determined by the decentering requirements! Generally, one or the other will be the more demanding on \( \Delta d \). It would appear that we should find which is the more stringent and use it to determine the tolerance allocations. The other would still make some contribution to the error budget, but not be independently determined. As in the tilt case, the ability of the element to roll to the full extent indicated in Eqn. 19 may be otherwise restrained by (but might also be caused by) a retainer ring, etc. Since this can be the case, it may be appropriate for the designer to use some judgement in the application of tilts and rolls after looking at the particulars of the sensitivities and the mounting designs to decide if they should be given the full force of the equations. This unfortunately seems to bring us back a bit from the engineering to the "art" in tolerancing.

Having now dealt with the necessary elements of the cost versus tolerances
and other parameters and the applicable geometric simplifications, we will next proceed with how we can apply the foregoing to distribute the tolerances for the maximum performance and cost benefit.

6. ASSIGNING TOLERANCES FOR MINIMUM COSTS, AN EXAMPLE

We described the general principles of distributing the tolerances for minimum cost in our first work (1) on this subject. Adams (6) made some significant additions to our work which we shall draw from below. If there is more than one performance criterion that must enter into the tolerancing process the solution to the equations is somewhat involved, but it can be done. However, many problems, including the one which we will use as an example, have one performance criterion which dominates all of the others as relates to the tolerances. That is, if the tolerances are chosen to meet that performance requirement, then all of the others will also be met. This reduces the computation considerably and makes it easier to visualize. For the balance of this paper, we will use the single requirement case with the understanding that it can be extended to multiple criteria by the methods of the previous papers as needed.

Figure 15 shows the Multi-Focal Length Tracking Telescope (MFLT) that we will use as an example of the tolerancing process. It has a catadioptric telescope section of 300mm aperture and about 2000mm focal length with a 25% central obstruction due to the secondary mirror. The telescope image is then collimated by a focus lens set. The afocal beam is then imaged by one of three relays to the final focal plane. These relays are alternately positioned in the beam to give the effective focal lengths of 1000, 2000, and 4000mm. Before the final focal plane there is an auto-iris system of variable neutral density filters and a reticle projection unit (AIR). There is also a 500mm system which is partially separated from the others to allow a larger field of view. The 500mm system is folded into the same optical path as the others by a movable prism. There are sealing windows in front of the telescope and the 500mm lens.

In this complex telescope system example, the most stringent requirement of the system is the on-axis MTF at 30 lp/mm. When this is satisfied, the off-axis MTF at 30 lp/mm, the on- and off-axis MTF at 10 lp/mm, and the boresight, etc. requirements will all also be satisfied without additional or adjusted tolerances being required. To be consistent with our previous report (1), we will designate this performance requirement by $E$ which represents the maximum permissible error in MTF from ideal for the system. We will actually convert this $E$ to units of RMS wavefront error (RMSWE) for simplicity. The total $E$ will eventually be partitioned among each of the tolerances which affect it.

To make a tractable example for this paper, we will partition the total $E$ among the various sections of the system. The partial $E$ will then be allocated to the parameter tolerances within one section based on the cost
FIGURE 15. EXAMPLE SYSTEM MULTI FOCAL LENGTH TRACKING TELESCOPE

FIGURE 16: DIFFRACTION MTF AT 30 lp/mm VS. WAVEFRONT ERROR FOR 2000mm, f/8 TELESCOPE
minimizing technique. This "divide and conquer" approach is needed here, plus any justifiable simplifications in general, to reduce the overwhelming magnitude of typical problems that have hundreds of component tolerances to be determined. The way to eat an elephant is one bite at a time!

In the final analysis, it is most correct to tolerance the whole optical train from object to image in one operation. This will truly allocate the tolerances for the required performance at the minimum cost. The simplifying partitioning will cause some deviation from the ideal result unless the estimate used in the partitioning was exactly correct. In the example used here, it would be best to tolerance the 4000mm from end-to-end, but the data would be too cumbersome to make a good illustration in this paper.

6.1 Simplifying approximations

The MTF of a system is often the best performance measure to use because it most directly relates in many cases to the performance of an overall system when it is used. It is, however, not generally possible to measure the MTF effect of each component lens of a system in the production process. The characteristics that are readily measured on a lens were discussed in Sec. 3, such as irregularity, radius, centration, etc. We chose to work here with the effects of each tolerance on RMSWE, because we think that it can be reasonably related to the system MTF.

We estimated the reduction in MTF per wave of RMSWE at 30 lp/mm for the 2000mm effective focal length, f/8 system by introducing errors into the system and evaluating it for MTF and RMSWE. With parameter deviations, we produced defocus, spherical aberration, coma, and astigmatism. Defocus was introduced by evaluating the system at different focal planes from the best focus. Spherical aberration was introduced by varying the y^4 aspheric coefficient from the nominal. Coma was evaluated in an "equivalent" (f/8, 2000mm) parabolic mirror system with the stop at the focal plane so that astigmatism was zero. The system was evaluated off-axis to introduce coma.

Lastly, astigmatism was introduced in an "equivalent" (f/8, 2000mm) Ritchy-Chretien telescope where coma and spherical aberration were zero. The system was evaluated off-axis to introduce astigmatism. The results appear in Fig. 16. All of the data forms a reasonably consistent pattern except the coma. We do not presently understand this anomaly which may be worthy of a separate study. However, since the effects of coma are less severe than the others, we will ignore them and use the conservative numbers indicated by the others.

Therefore we will use delta RMSWE = delta MTF/.60 as the amount of reduction in MTF that will be accompanied by a corresponding RMSWE. This will allow us to work with the effect of tolerances on the RMSWE which we will assume are quasilinear in the regions where we are applying them. This may be a conservative estimate, but we would like to err on that side.
Another approximation that we will draw upon comes from Smith (10) where

\[ \text{RMS} = \frac{\text{Peak-to-valley}}{3.5} \]  

(20)

approximates the RMSWE expected from most types of error. It would seem that sharp departures over a small portion of the wavefront would violate this rule, but those are not usually encountered. We made a small investigation of our own by comparing the RMS and P-V data on many interferograms from a ZYGO interferometer. This leads us to think that the factor in Eqn. 20 might be more like 7 than 3.5 when small irregularities such as those on surfaces intended to be spheres are examined. For this example, however, we will use Smith's value.

In the example design, the apertures were selected at the first order stage to yield the required MTF when the diffraction effects of the obstruction plus one quarter wave of design and fabrication errors were taken into account. This is not much error to spread across the many elements from the object to the focal plane. One help is the fact that certain compensating alignments can be made at assembly since the systems will not be made in very large quantities. We will use the approximation of Eqn. 20 to establish a preliminary total error budget of 0.071 RMSWE (1/4 wave P-V) from all sources in laboratory tests. In the final application, obviously, atmospheric and other effects might influence the results further.

6.2 Error budgets

We need to now decide how to distribute this 0.071 RMSWE among the many facets and tolerances of the system. Smith (10) describes how to work with the root-sum-of-the-squares (RSS) to combine error effects. McLaughlin (14) shows that RSS tends to be too pessimistic and Smith (10) himself concludes that it may err on the conservative side. McLaughlin shows that the total system error will tend to be 0.42 times the RSS prediction if the fabrication errors have a Gaussian distribution which is truncated at the 2 sigma level. Although there is a major move at this time in industry to apply 6 sigma tolerancing, we believe the 2 sigma to be appropriate in this case where individual adjustment and testing is required. We will therefore use McLaughlin's 0.42 factor for the fabrication errors.

To simplify the example, we will partition the 4000mm path of the system. In looking at Fig. 15, we count 32 surfaces through the 4000mm optical path. The authors chose to emphasize the sensitivity effects of mirrors by counting them twice to give 34 as the surface count. Of this 34, 8 are in the telescope, 8 in the focus system, 12 in the 4000mm relay, and 6 in the AIR. The other paths are less complex. This one will be the critical path and set the pace for the telescope, focus system, and AIR tolerances. We will allocate the budget to the four sections of the 4000mm path (telescope, focus, relay, AIR) in proportion to the square root of the number of surfaces in the section divided by the total number of surfaces. This is an engineering estimate of the relative influence of each section.
The division of the system into these sections is also because they can be tested independently by section for RMSWE in production. Figure 17 shows the error budget broken down this way. The top level requirement was determined above to be effectively .071 RMSWE. We know from the design stage that the design has used up .030 RMSWE. Another analysis indicates that the effects of alignment focus errors and the laboratory environment should be on the order of .009 RMSWE. This leaves .0637 RMSWE to RSS with the other two (three) parts of that level of the budget to give .071 RMSWE.

From McLaughlin's information and the assumption of Gaussian errors, we then divide the fabrication budget of .0637 by .42 to give 0.1517 RMSWE which can be distributed over the four sections of the 4000mm system. The bottom four boxes of the budget in Fig. 17 show how these work out when the above argument is applied. We will work through the simplest section, the focus lens with .0736 RMSWE budget, as an example of the procedure for tolerance distribution to give the minimum cost while meeting the performance requirements (to within some statistical uncertainty).

6.3 Derivatives of costs with respect to tolerances

In the assignment of tolerances for minimum cost as we will show below, it is necessary to have the partial derivatives of the total cost with respect to the reciprocal of each tolerance. These are basically derived from Eqn. 11 and the metal tolerance costs in Fig. 12. Since we showed that the metal diameter and lens centering tolerances can be made dependent on the glass diameter tolerance, we only have five types of tolerances to allocate in the framework which we have been discussing. These are irregularity, radius, thickness, diameter, and tilt of a lens due to the errors of the cell. Roll is shown in Fig. 14b and can be derived from delta d. Tilt is just the cell run-out parallel to the optical axis (delta LE of Fig. 12) divided by the diameter d of the lens. We will call the partial derivatives of the total system cost with respect to the reciprocal of these tolerances $I, $R, $T, $d, and $L. The $L applies to both the metal bore diameters and axial run-out. The first three will be functions of the base grind and polishing costs with a common factor that we will call BP which is defined in Eqn. 21.

$$BP = P \times Y \times 14 / N^#$$

(21)

The $N^#$ is the $N$ for the given surface number just as we will use delta $I^#$, delta $R^#$, etc. for those values associated with that surface number. The $d$ has as a factor of the base centering cost $CE$ while $L$ has as a factor the base machining cost $MF$. There are additionally three "fudge" factors $FI$, $FR$, and $FT$ associated with $I$, $R$, and $T$ which can be taken as unity for a simplifying assumption or calculated in each case as we will explain below.

Equations 22 through 26 give these partial derivatives which will be needed to allocate tolerances.

$$I^# = BP \times FI \times .25 \times KI$$

(22)

$$R^# = BP \times FR \times 8 \times KZ$$

(23)
FIGURE 17. ERROR BUDGET ALLOCATION, 4000mm EFL OF MULTI-FOCAL-LENGTH TRACKING TELESCOPE

FIGURE 18. FOCUS LENS SET EXAMPLE
The factors FI, FR, and FT come from taking the partial derivatives of Eqn. 11 with respect to the reciprocal of the tolerances. They are due to the modifying effects of other parameters and are given in Eqns. 27, 28, and 29.

\[
FI = (1 + 8KZ*(R#/d)^2 / (\text{delta } R#) + 0.0003 \times (d/T)^2)
\]  
(27)

\[
FR = (1 + 0.25 \times KI / (\text{delta } I#))
\]  
(28)

\[
FT = (1 + 10/S# + 5/D#) \times (1 + 0.01 \times SC^{-3})
\]  
(29)

There is one problem with Eqns. 27 and 28, however. The values of delta R# and delta I# are not defined until after the tolerancing process. We can include the effects of FT in Eqn. 24 because all of its parameters are known at the start. We chose to approximately include the effects of FI and FR after the tolerances are calculated by dividing the resulting tolerances for irregularity and radius by the cube roots of FI and FR respectively. The reasons for this will be more apparent from Eqn. 32 in Sec. 6.4. The other alternative would be substituting FI and FR back in an iterative procedure.

We now only need to address the simple allocation equations and procedures and we can finish the task of finding the least cost tolerances.

6.4 Tolerance allocation process

We showed previously (1) that the total error E was the sum of all of the contributions from each error source which is the product of that tolerance value t(i) times the sensitivity S(i) of the performance to variations of that parameter as seen in Eqn. 30

\[
E = \text{sum over all i} (S(i) \times t(i))
\]  
(30)

We showed that the tolerances could be distributed for minimum cost by applying Eqn. 31 to each tolerance in turn. The A(i)'s are the coefficients of the derivatives of cost given in Eqns. 22 through 26. In Eqn 31, the sum of all the square roots of the products of the cost coefficients A(k) and the sensitivities S(k) is divided into the total error budget E to get a constant which is multiplied by a function of the cost and sensitivity of each tolerance.
Adams (6) pointed out that Eqn. 31 gave the solution which represented the tolerances all going to the worst case limit condition, and this is obviously too severe. He showed that the RSS condition would be satisfied by the tolerance distribution if Eqn. 32 was applied instead. We are indebted to Adams for this contribution and will apply it to the problem at hand as the most appropriate solution.

\[
t_i = \frac{\sqrt{A_i}}{|S_i|} \frac{E}{\sum_{K=1}^{n} \sqrt{A_K |S_K|}}
\]  

\[
t_i = 3 \frac{\sqrt{A_i}}{S_i^2} \frac{E}{\sqrt{\sum_{K=1}^{n} 3 (A_K S_K)^2}}
\]

The allocation is then very straightforward in the single \( E \) case. It is only necessary to develop a table of \( A(i) \) and \( S(i) \) for all appropriate \( i \) and process the data in accord with Eqn. 32. Table 3 gives these values for the example case and the resulting \( t(i) \) which we seek are in Table 4.

The example case has a total of four lenses which we illustrate in Fig. 18. The field lens is close to the focal plane and has negligible sensitivity. We therefore remove it from consideration since we can assign minimum cost tolerances to it without affecting the rest of the task. This then reduces the example to a three element lens where the surfaces and dimensions are numbered as in Fig. 18.

The necessary data processing is very conveniently set up and done using a spreadsheet program for data entry and all of the necessary calculations. Table 3 starts with the parameters determined from the "lens drawing" or design parameters. The number per block for each side is determined from Eqns. 4 and/or 5. The sensitivities of the system performance to each parameter are determined from the lens design program. The base costs and cost versus the reciprocal tolerance derivative coefficients are calculated from Eqns. 22 to 26. The constant multiplier in Eqn. 32 is calculated. The individual factors from the individual sensitivities and cost derivatives are used to compute the tolerances for each of the tolerated parameters. The adjustments to the I and R tolerances are made for FI and FR as mentioned in the previous section.

Table 4 contains the resulting tolerances for which we have been working. It is the set of tolerances for each of the tolerated parameters which will give the least cost solution and meet the performance with some statistical "RSS" certainty. The assumption per Adams (6) is that the errors will be distributed about the norm in a Gaussian manner and the tolerance limits
TABLE 3
TOLERANCE DATA & COMPUTATIONS

E = 0.0736 RMSWE

<table>
<thead>
<tr>
<th>EXAMPLE FOCUS LENS SET</th>
</tr>
</thead>
<tbody>
<tr>
<td>SURFACE #</td>
</tr>
<tr>
<td>RADIUS</td>
</tr>
<tr>
<td>DIAMETER</td>
</tr>
<tr>
<td>THICKNESS</td>
</tr>
<tr>
<td>SCRATCH</td>
</tr>
<tr>
<td>DIG SPEC.</td>
</tr>
<tr>
<td>REFRACTIVE INDEX</td>
</tr>
<tr>
<td>STAIN #</td>
</tr>
<tr>
<td>POLISHABILITY</td>
</tr>
<tr>
<td>CHAMFER</td>
</tr>
<tr>
<td>FLATS</td>
</tr>
<tr>
<td>NUMBER/MILLING LOT</td>
</tr>
<tr>
<td>CENTERING LOT SIZE</td>
</tr>
<tr>
<td>YIELD</td>
</tr>
<tr>
<td># PER BLOCK</td>
</tr>
<tr>
<td>KI (fringes)</td>
</tr>
<tr>
<td>KZ (sag, inches)</td>
</tr>
<tr>
<td>KT (thk, inches)</td>
</tr>
<tr>
<td>KD (dia, inches)</td>
</tr>
<tr>
<td>KM (met dia, in)</td>
</tr>
<tr>
<td>KW (mrads wedge)</td>
</tr>
<tr>
<td>SI dRMS/FRINGE</td>
</tr>
<tr>
<td>SZ dRMS/INCH</td>
</tr>
<tr>
<td>ST dRMS/INCH</td>
</tr>
<tr>
<td>Std dRMS/INCH</td>
</tr>
<tr>
<td>SLE dRMS/radian</td>
</tr>
<tr>
<td>f, CLEARANCE</td>
</tr>
<tr>
<td>MF (MFG BASE)</td>
</tr>
<tr>
<td>BP (EQN 21)</td>
</tr>
<tr>
<td>CE (EQN 3)</td>
</tr>
<tr>
<td>A$I (EQN 22)</td>
</tr>
<tr>
<td>A$Z (EQN 23)</td>
</tr>
<tr>
<td>A$T (EQN 24)</td>
</tr>
<tr>
<td>A$&amp; (EQN 25)</td>
</tr>
<tr>
<td>A$L (EQN 26)</td>
</tr>
<tr>
<td>(A(K)*S(K))$^{2/3}$ I</td>
</tr>
<tr>
<td>(A(K)*S(K))$^{2/3}$ Z</td>
</tr>
<tr>
<td>(A(K)*S(K))$^{2/3}$ T</td>
</tr>
<tr>
<td>(A(K)*S(K))$^{2/3}$ d</td>
</tr>
<tr>
<td>(A(K)*S(K))$^{2/3}$ L</td>
</tr>
<tr>
<td>CONSTANT E/SQR(SUM)</td>
</tr>
<tr>
<td>(A(I)/S$^2$)$^{1/3}$ I</td>
</tr>
<tr>
<td>(A(I)/S$^2$)$^{1/3}$ Z</td>
</tr>
<tr>
<td>(A(I)/S$^2$)$^{1/3}$ T</td>
</tr>
<tr>
<td>(A(I)/S$^2$)$^{1/3}$ d</td>
</tr>
<tr>
<td>(A(I)/S$^2$)$^{1/3}$ L</td>
</tr>
<tr>
<td>delta I (fringes)</td>
</tr>
<tr>
<td>delta Z (inches)</td>
</tr>
<tr>
<td>delta AT(tilt,rd)</td>
</tr>
</tbody>
</table>
### TABLE 4
RESULTING TOLERANCES AND COSTS

#### TOLERANCES:

<table>
<thead>
<tr>
<th>SURFACE #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta R (inches)</td>
<td>0.003452</td>
<td>0.006652</td>
<td>0.002920</td>
<td>0.344788</td>
<td>0.001924</td>
<td>0.000851</td>
</tr>
<tr>
<td>delta I'(fringes)</td>
<td>0.288933</td>
<td>0.344375</td>
<td>0.128449</td>
<td>0.140610</td>
<td>0.249586</td>
<td>0.298252</td>
</tr>
<tr>
<td>delta T (inches)</td>
<td>0.004804</td>
<td>0.000712</td>
<td>0.001960</td>
<td>0.003111</td>
<td>0.001250</td>
<td>0.594910</td>
</tr>
<tr>
<td>delta AR(roll,rd)</td>
<td>0.000013</td>
<td>0.000007</td>
<td>0.000008</td>
<td>0.000000</td>
<td>0.000051</td>
<td>0.000062</td>
</tr>
<tr>
<td>delta td (inches)</td>
<td>0.000070</td>
<td>0.000098</td>
<td>0.000231</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>delta LE (inches)</td>
<td>0.000176</td>
<td>0.000118</td>
<td>0.000689</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>delta d (inches)</td>
<td>0.000041</td>
<td>0.000050</td>
<td>0.000106</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>delta dM (inches)</td>
<td>0.000052</td>
<td>0.000063</td>
<td>0.000135</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>delta A (mrad.)</td>
<td>0.001419</td>
<td>0.001715</td>
<td>0.003611</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>delta A ('dev)</td>
<td>0.002442</td>
<td>0.003541</td>
<td>0.006212</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Metal Dia.=dM</td>
<td>2.350094</td>
<td>2.310114</td>
<td>2.040241</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### BASE COST COMPUTATION

<table>
<thead>
<tr>
<th>LENS #</th>
<th>1-2</th>
<th>3-4</th>
<th>5-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>MILLING = MG</td>
<td>17.40939</td>
<td>17.39075</td>
<td>17.27330</td>
</tr>
<tr>
<td>GR &amp; POL = GP</td>
<td>39.57352</td>
<td>23.4</td>
<td>42.46666</td>
</tr>
<tr>
<td>CENT &amp; EDGE=CE</td>
<td>7.497619</td>
<td>7.484285</td>
<td>7.394285</td>
</tr>
<tr>
<td>BASE COST TOTAL =</td>
<td>64.48053</td>
<td>48.27503</td>
<td>67.13425</td>
</tr>
</tbody>
</table>

#### COMPUTATION OF TOTAL LENS COST = MT

<table>
<thead>
<tr>
<th>LENS #</th>
<th>1-2</th>
<th>3-4</th>
<th>5-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>MILLING/GENERATE</td>
<td>17.40939</td>
<td>17.39075</td>
<td>17.27330</td>
</tr>
<tr>
<td>PART SETUP</td>
<td>21</td>
<td>18.2</td>
<td>18.2</td>
</tr>
<tr>
<td>CENTERING</td>
<td>795.6429</td>
<td>658.6013</td>
<td>312.9745</td>
</tr>
<tr>
<td>TOTAL LENS COST</td>
<td>1556.309</td>
<td>717.6634</td>
<td>403.9732</td>
</tr>
</tbody>
</table>
However, a significant problem appears in Table 4. Many of the tolerances are well beyond what can be achieved in normal practice; they are off the chart in Figs. 2 to 12! This is a disappointing result for the designer, but hopefully not the end of the road. Finding the problem at the design stage is not nearly as frustrating or expensive as finding it at the production stage.

The lower part of Table 4 shows the application of the above formulas to compute the base cost for each of the lenses and the total costs when the tolerances are included. The latter are about 15 times that of the easy base lens if they could even be made. "Off-the-chart" implies in most cases that it cannot be made or at least it would be much more expensive than the linear chart data would predict. We may have therefore identified here an impractical design. The designer then has the challenge of finding a design and/or an approach which will be less sensitive. The addition of more lens elements is not at all out of the question if they can reduce the sensitivity enough. If the added cost of one or more elements reduces the tolerance costs sufficiently and all the lenses can be built, the total cost will be less than the first design. It might be possible to cement a doublet to get rid of a sensitive air-space. It might be practical to make a centering adjustable and/or add other assembly tricks (15). The designer can now evaluate the impact of design changes on cost by using the tools put forth in this paper. The "bottom line" in Table 4 can tell him if he has improved the situation or not.

What we see here is an example of the processes in Fig. 2 where the tolerance sensitivity analysis and distribution feeds into the producibility analysis which sends us back to the detail optical design for further work.

A complex system such as the whole 4000mm example system will have several times as many columns as Table 3, but the process is the same and relatively straightforward to apply. The most difficult aspect can be obtaining the sensitivities of the performance criterion to parameter variations. Existing lens design programs can do this with greater or less facility, but all should be readily modifiable to straightforwardly generate the data needed. This data generation is computer intensive and time consuming, but probably unavoidable. The tolerancing program described above takes only a few seconds on a PC to calculate the tolerances for the six surface case. It should be approximately linear with number of surfaces as long as only one performance criterion is to be considered. Multiple criteria would be more cumbersome to evaluate as we showed in our first work (1). However, Adams (6) shows that it is most likely that one criterion is all that is needed, and more than two is highly unlikely.
7. SUMMARY AND CONCLUSIONS

We had previously (1,2,3) shown the principles of how to assign tolerances to give the minimum production cost and we mentioned the possible application of the results to estimating total lens production cost. In this work, the previous data and principles have been refined and some of the results provided by Adams (6), Smith (10), Parks (8), and Fischer (7) have been incorporated. The lens cost estimating formulas mentioned in the earlier work (3) have been developed into useful tools. A new analysis is presented of the interdependency of the lens and cell diameter tolerances as a result of the cost versus tolerance knowledge. And finally, a minimum cost tolerancing procedure has been reduced to practice in a form which is straightforward and accurate enough for practical engineering application.

It is now practical to estimate the production cost of most lenses by entering the drawing data into a spreadsheet program. This is essentially an "expert system" estimator which can be applied by one with very little training or experience to get as good or better cost estimates for most lenses than an expert. It is also now practical to distribute lens tolerances using a spreadsheet program such that the production costs are minimized and to find the impact of design changes on lens costs. Both of these tools have advanced what was an "art" to an engineering discipline. Both are more accurate than what has been the common practice, and they are more accurate than the cost data as it typically is measured at this time.

The application of the estimating program can reduce the business overhead cost of a production operation, and it can point to the cost drivers of any particular lens such as: setup, milling, polishing, or centering. The application of the tolerancing program can decrease the production cost of systems from what they typically have been in both lens production, assembly and testing. If the tolerances are unnecessarily tight, the lens production cost is wasteful. If the tolerances are not tight enough to give a good yield of deliverable parts, the assembly and testing costs are wasteful. This tool attacks the economic problem at the point of the most potential impact on the life cycle costs as shown in Fig. 1, the detail design and analysis phase.

8. ACKNOWLEDGEMENTS

The author gratefully acknowledges the extensive contributions and discussions of Reinhard Seipp and Steven J. Cupka on the subject of base cost and cost versus tolerances and the assistance of David Poole and Alice Covington in the preparation of the figures.

9. REFERENCES


