Effect of Substrate Index of Refraction on the Design of Antireflection Coatings

Ronald R. Willey
Willey Optical, Consultants, 13039 Cedar Street, Charlevoix, MI 49720

1. INTRODUCTION

Formulae to estimate the average percent reflectance (Rave) of a broadband antireflection (AR) coating as a function of the bandwidth (B), the overall thickness (C), the index of refraction of the last layer (L), and the difference between the indices of the high- and low-index layers (D) were reported in 1991[1]. Various refinements of these formulae and other insights into the underlying behavior of such coating designs have been reported up until the present time[2-5]. Dobrowolski, et al.[6] and Tikhonravov, et al.[7] have also added independent viewpoints to this subject over this period. In the previous studies, the effects of the index of refraction of the substrate have mostly been ignored and have appeared to be very minor. This study has investigated the influence of the substrate index on the Rave results. It has been found that there seem to be two classes of designs with respect to the effect of substrate index. In the class of "step down"[8,9] AR designs, there is a significant effect, in the other class, there is no significant effect. Even in the step-down case, there is no effect of substrate index if any and all indices of refraction for the coating materials are available from that of the index of the substrate to the index of the media.

2. IDEAL AR COATINGS

It is known mathematically that the Fourier transform of a Gaussian function is again a Gaussian function. In the case of an optical coating, a Gaussian index versus thickness profile will transform and produce a reflectance versus frequency profile which is also Gaussian. The equation for such an index profile is shown in Eqn. 1 where $\mathbf{n_s}$ is the substrate index, $\mathbf{n_i}$ is the index of the medium (air/vacuum), and N(x) is the index versus thickness, x.

$$\boldsymbol{N(x) = n_i + (n_s - n_i)e^{-x^2}} \tag{1}$$

* ron@willeyoptical.com; phone 1 231 237-9392; www.willeyoptical.com

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This Gaussian function rapidly goes from $\mathbf{n_s}$ to very nearly $\mathbf{n_i}$ (to within >1% of the difference $\mathbf{n_s - n_i}$) with an x of approximately 2.15. The reflectance spectrum correspondingly goes from that of the substrate to near zero over a small range of frequency from 0 to some lowest frequency in the AR band (longest wavelength), and then the reflectance is essentially zero from there to an infinite frequency (very short wavelength). This therefore provides an almost semi-infinite AR band. Figure 1 shows the reflectance versus frequency (in wavenumber (cm^{-1})) for four (4) such index profiles from substrates of index 4.0, 3.0, 1.865, and 1.5. The longest AR wavelength in this case is approximately 6000 wavenumbers or 1667 nm.

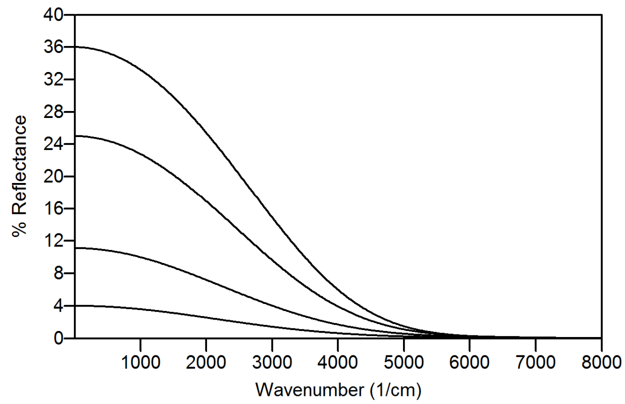

Figure 1. Reflectance versus frequency for ideal Gaussian index of refraction profiles on substrates of index 4.0, 3.0, 1.865, and 1.5.

Figure 2 shows the same designs as in Fig. 1 on a very broad band where B equals 16.7 (highest frequency divided by the lowest, 100000/6000 = 16.7). This alludes to the semi-infinite nature of the AR band.

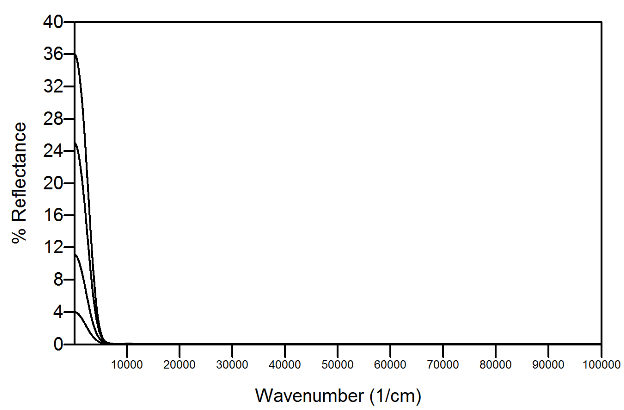

Figure 2. Reflectance versus frequency for ideal Gaussian index of refraction profiles as in Fig. 1 on substrates of index 4.0, 3.0, 1.865, and 1.5 over a very broad spectral range.

The cases illustrated in Figs. 1 and 2 all form "perfect" AR designs on all substrate indices. Therefore, if all indices from $\mathbf{n_s}$ to $\mathbf{n_i}$ are available, there is **no effect** of substrate index on what can be achieved in Rave.

However, the above is predicated on the availability of all indices from $\mathbf{n_s}$ to $\mathbf{n_i}$, and that there is no absorption in the materials (k-values are all zero). Such materials are not available in the real world. It has been shown[10] that the real-world performance is limited by the lowest real index available; and the reflectance over a very broad band will be approximately the same as if the substrate and coating was just a slab of the index of that lowest index material.

3. LESS THAN IDEAL STEP-DOWN AR COATINGS

The limitation of real-world indices becomes the issue to deal with in attempting to approach in the real world what can be done in an ideal world. If a high index coating material similar to that of the substrate is available, it can be used with the lowest practical real-world index available for that spectral region of interest. The techniques of Herpin[11], Epstein[12], and Ohmer[13] can be applied to approximate the ideal index profile down to that of the lowest real index. Figure 3 shows such an index profile, where the ideal profile has effectively been divided into seven pieces which have been replaced by sets of layers of high-low-high index which approximate the index of that piece of the ideal profile. Figure 4 then shows the reflectance versus thickness of that design at the longest wavelength in the AR band (6250 cm^{-1}).

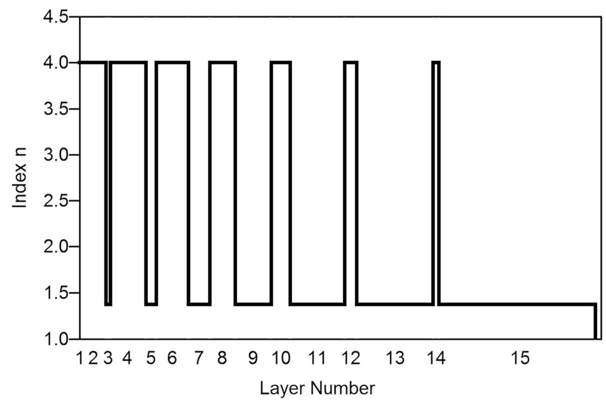

Figure 3. Index versus thickness of an approximation of the ideal step-down profile by layers of homogeneous H = 4.0 and L= 1.38 on a substrate of index 4.0.

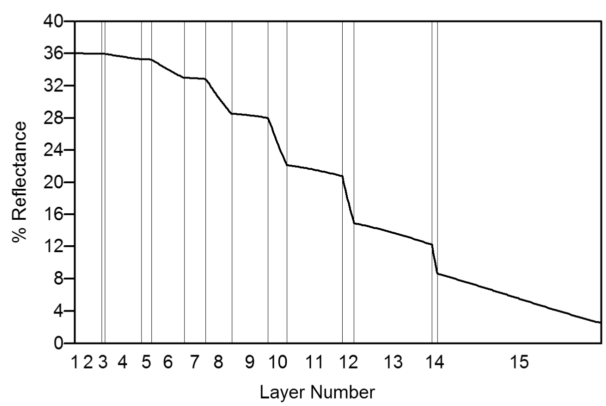

Figure 4. Reflectance versus thickness at the lowest frequency (longest wavelength) in the AR band (6250 cm^{-1}) for the step-down of Fig. 3.

Figure 5 shows the spectrum for this design as the curve which starts at 36% reflectance, and it has the lowest in the broad AR band from 6250 to 37500 cm^{-1} (B = 6). The other curves are the results of using the same design technique on substrates of lower indices. In these designs, the layers are H = 4 and L = 1.38. It can be seen that step-down designs result in each case, except when the substrate index is below some threshold. The reflectance Rave in the AR band also becomes higher with decreasing substrate indices. In these cases, **the resulting Rave is a function of the substrate index**. This would seem to be due to the fact that it is not possible to approximate the ideal index profile well enough; the ideal step-down results as seen in Figs. 1 and 2 do **not** depend on the substrate index.

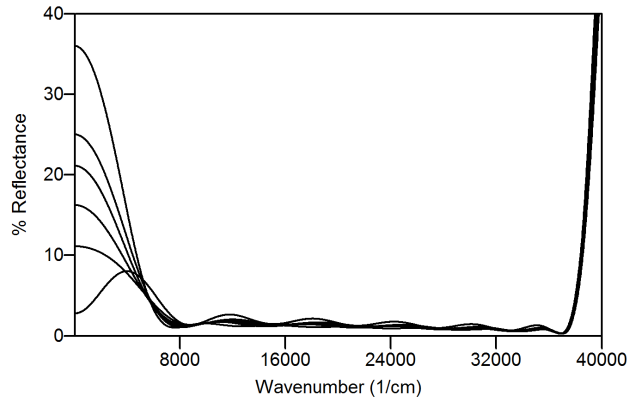

Figure 5. Reflectance versus wavenumber in the AR band 6250-37500 cm^{-1} for the step-down of Fig. 4 and for substrates of lesser indices, down to below a threshold which results in a non-step-down design.

All of the above step-down designs have thicknesses which are approximately 1.3 to 1.4 quarter wave optical thickness (QWOT) at 6250 cm^{-1} (longest wavelength in the AR band). The following section will deal with designs that are thicker than this by factors of about 2 to 6.5 times; these are in a different class than step-down designs.

4. NON-STEP-DOWN AR COATINGS

In a recent report on broad band AR designs[4], it was again shown that designs tend to gravitate into quantized overall thicknesses. Figures 6 and 7 are reproductions of two figures from that report which illustrate the likely configurations which might result in optimal designs depending on the overall thickness and the substrate index of the designs. Figure 6 shows optimized designs on a high index substrate (2.35) which result from different overall coating thicknesses from the step-down which is about ½ cycle to non-step-downs up to 3-1/2 cycles. These cycles are related to what Dobrowolski, et al.[6] and Tikhonravov, et al.[7] have called "clusters'. The optical thicknesses of these cycles are approximately 2.5 QWOTs at the longest wavelength in the AR band per cycle, plus 0.37 QWOT. Therefore, in Fig. 6, these would be approximately 1.62, 4.12, 6.62, and 9.12 QWOTs thick. The coating materials in Figs. 6 and 7 are H = 2.35 and L = 1.46. As the designs get thicker the design, the Rave gets lower, as reported in previous papers[1-5].

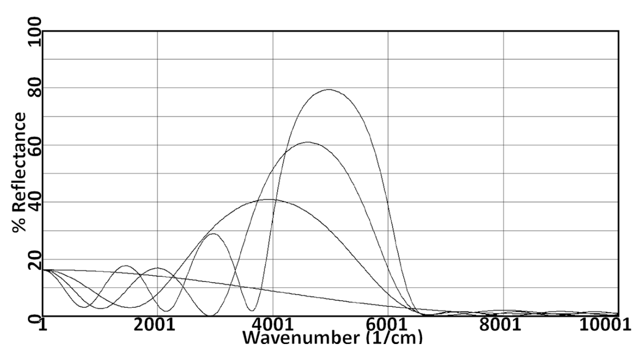

Figure 6. Reflectance versus wavenumber of optimized designs on a high index of refraction substrate (2.35), showing a step-down (1/2 cycle), 1-1/2 cycle, 2-1/2 cycle, and 3-1/2 cycle design.

Figure 7 shows optimized designs on a low index substrate (1.52) which result from different overall coating thicknesses from one (1) cycle to three (3) cycles.

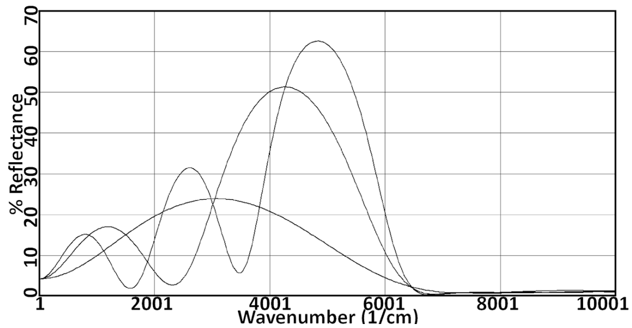

Figure 7. Reflectance versus wavenumber of optimized designs on a low index of refraction substrate (1.52), showing a 1, 2, and 3 cycle design.

Figure 8 shows one cycle designs on substrates of index 2.35, 2.108, 1.865, 1.623, and 1.38 which have been optimized over the band of 6250 to 25000 cm^{-1} (B = 4). Note that the design for the highest index substrate tends to a step-down, but the rest of the designs tend to the one cycle form.

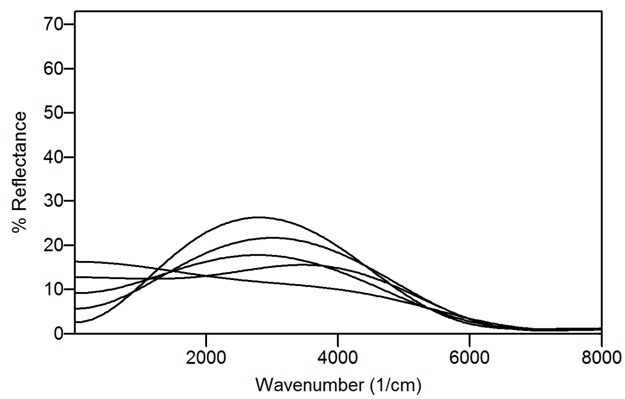

Figure 8. Reflectance versus wavenumber of optimized one (1) cycle designs on substrates of index 2.35, 2.108, 1.865, 1.623, and 1.38 over a spectral bandwidth of 4 and average thickness of 2.87 QWOTs at 6250 cm^{-1}.

Figure 9 shows designs of two cycle thickness over the same range of substrate indices as Fig. 8.

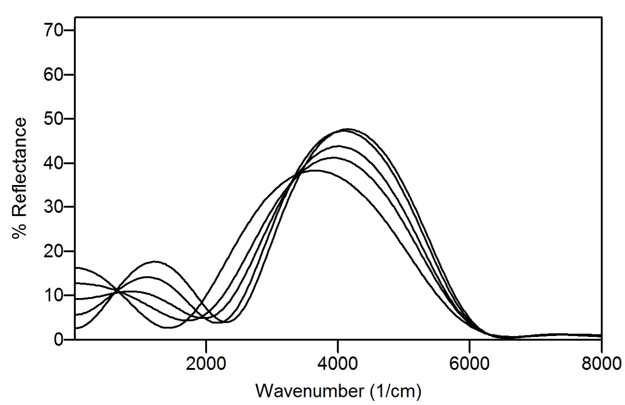

Figure 9. Reflectance versus wavenumber of optimized two (2) cycle designs on substrates of index 2.35, 2.108, 1.865, 1.623, and 1.38 over a spectral bandwidth of 4 and average thickness of 5.22 QWOTs at 6250 cm^{-1}.

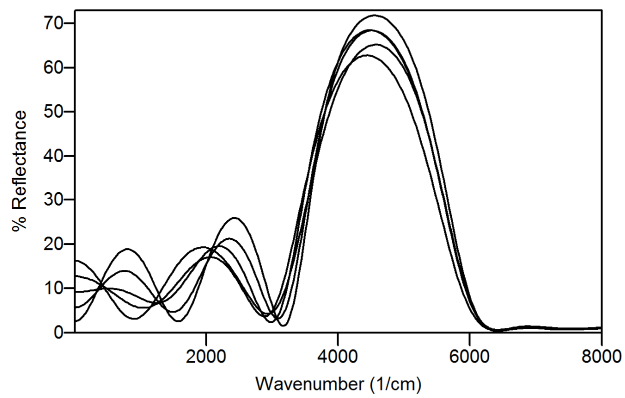

Figure 10. Reflectance versus wavenumber of optimized three (3) cycle designs on substrates of index 2.35, 2.108, 1.865, 1.623, and 1.38 over a spectral bandwidth of 4 and average thickness of 7.87 QWOTs at 6250 cm^{-1}.

Figure 10 shows designs of three cycle thickness over the same range of substrate indices as Figs. 8 and 9.

All of the designs in Figs. 8 to 10 result in Rave values that are essentially **not** a function of substrate index as summarized in Fig. 11. The Rave **is** a function of the overall thickness as reported earlier[1-5] and seen in Fig. 11, but Rave **is not** a function of substrate index of refraction except for the step-down class of designs.

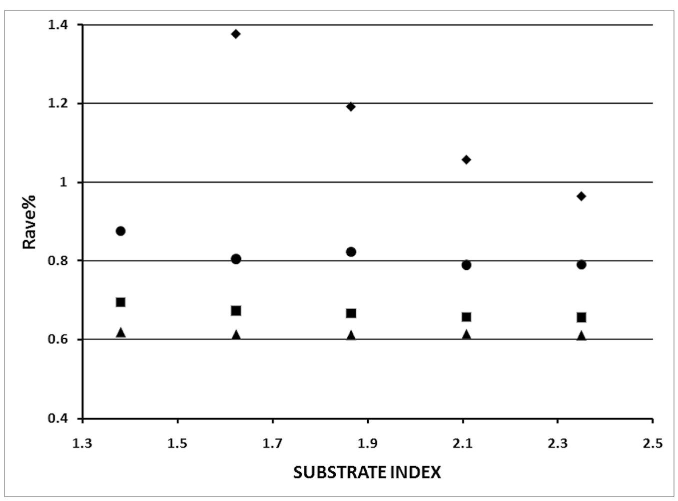

Figure 11. Average reflectance in the AR band of B = 4 versus substrate index of refraction for ½ cycle (step-down) designs (diamonds), one cycle designs (round dots), two cycle designs (squares), and three cycle designs (triangles).

5. SUMMARY AND CONCLUSIONS

It has been shown that average reflection in a broad AR band is **not** a significant function of substrate index of refraction **except** in the step-down class of designs. The step-down class is identified as having an overall thickness which is less that approximately 1.5 QWOTs at the longest wavelength in the AR band. Thicker designs are not a significant function of substrate index. Designs from 2.5 to 9 QWOTs overall thickness appear to be able to compensate or adjust for any substrate index by the design of the earlier layer structure of the AR coating, as long as the high index of the coating is equal to or greater than that of the substrate.