Cassegrain-Type Telescopes

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RECENTLY, the writer decided to compare the performance to be expected of a Cassegrain telescope with those of various other systems which logically represent types evolved from it. The study was made in the course of developing an automatic lens-design program for use on the IBM 7090 computer at United Aircraft Research Laboratories. Here we shall consider the Cassegrain, Dall-Kirkham, Ritchey-Chrétien, Maksutov-Cassegrain, and Schmidt-Cassegrain systems.

In the computing program[1] used, we can trace the paths of light rays through any system of lenses and mirrors that is symmetrical with respect to the optical axis. These optical elements may be spherical or aspherical. Any one of the lens or mirror surfaces can be figured mathematically to bring all of the rays to a point focus on axis. Then rays are traced from some off-axis object point (which might represent a star at infinity) through a rectangular array of equally spaced points in the entrance pupil of the system, and on through the subsequent refracting and reflecting surfaces until they intersect some plane at which we wish to examine the pattern formed by the rays; for example, this may be the principal focal plane.

If the lens were perfect, all the originally parallel rays would pass through a single point (neglecting diffraction effects) in the appropriate plane. Since the lens cannot be perfect but suffers from aberrations, the rays will pass through several points, forming the patterns shown by the spot diagrams found in Figs. 4 and 6.

The larger the pattern, the greater the aberrations and the less the inherent resolution of the optical system. Since the rays pass through the centers of squares of equal area in the entrance pupil, the density of points in the spot diagram directly indicates the light-intensity distribution in the image, again neglecting diffraction. Therefore, we can expect the image of a star seen through the instrument with a good eyepiece to look like the computed spot diagram if we mentally smooth the points into a continuous density distribution. Of course, where the points are widely separated, that part of the image may be too faint to be seen or photographed.

On command, the computer can find the focal planes where the image has the smallest spread in the X′ direction of Fig. 1, the Y′ direction, and the smallest overall spread. These are respectively the sagittal-focus, tangential-focus, and circle-of-least-confusion planes (Fig. 2).

The whole lens system and its object and image points have mirror symmetry about the tangential plane, which contains the image point and the optical axis of the system. It is, therefore, only necessary to trace rays through one half of the entrance pupil, as shown in Fig. 1, in order to obtain all the information available from the computation. Each spot diagram therefore represents only half the system's aperture — the remaining half can be included by visualizing the diagram's mirror image about the horizontal line indicated in each case.

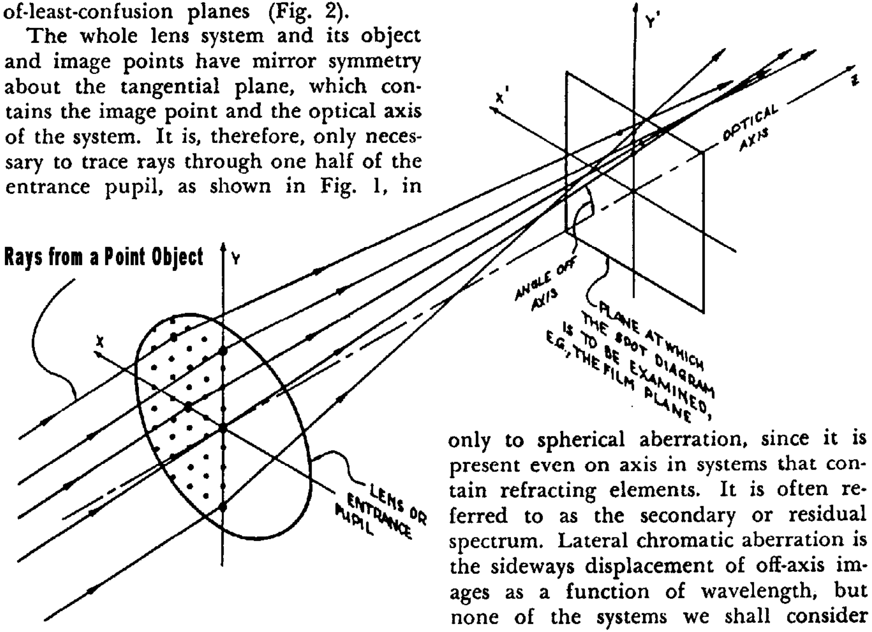


Fig. 1 (above). An isometric schematic view of ray paths through the entrance pupil of an imperfect lens, producing a spot diagram in the selected observing plane.

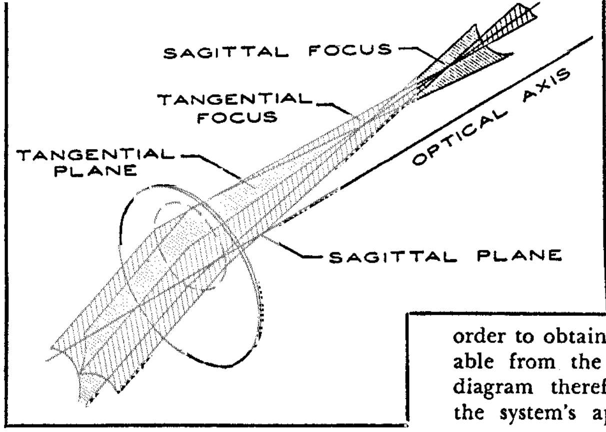


Fig. 2 (left). The relation between the sagittal and tangential planes, the latter containing the system's optical axis.

In astronomy, we are generally concerned with five primary monochromatic aberrations and two chromatic ones. *Spherical aberration* deteriorates all images, including on-axis ones. *Coma* decreases the resolution of image points, even those only slightly off axis. *Astigmatism* affects image points somewhat more off axis. *Curvature of field* requires photographic emulsions to be curved if off-axis images are to remain in focus.

The fifth monochromatic aberration is *distortion*, which is usually neglected by astronomers, except when measuring star fields. As distortion can be allowed for mathematically, we shall not consider it.

The primary color aberrations are *longitudinal* and *lateral* chromatic aberration. The first occurs on axis, causing focal length to change with wavelength; it produces the colored halo commonly seen in refractors but absent in reflectors. This aberration is second in importance only to spherical aberration, since it is present even on axis in systems that contain refracting elements. It is often referred to as the secondary or residual spectrum. Lateral chromatic aberration is the sideways displacement of off-axis images as a function of wavelength, but none of the systems we shall consider suffers significantly from this effect.

Planetary observers, because of the very small fields of view that they require, are concerned almost exclusively with spherical aberration and longitudinal color. Lunar observers, studying an object half a degree in diameter, are bothered by coma and possibly by astigmatism if it is severe. Those photographing fields of stars and nebulae will be concerned with these four aberrations plus curvature of field; they must either compensate for a curved field by bending their emulsions, as in a Schmidt, or use a field-flattening lens.

Aside from any consideration of aberrations, the advantages of a compound system such as a Cassegrain come mainly from the short tube length. This reduces flexure problems, permits building a smaller observatory, provides greater

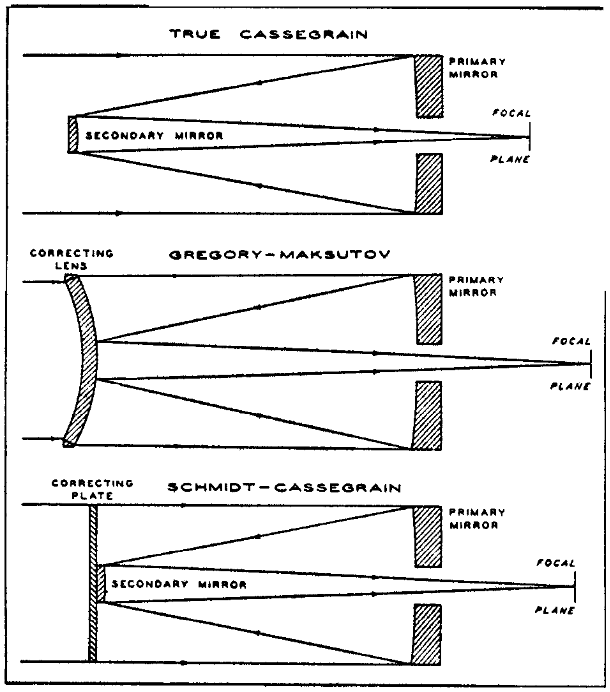
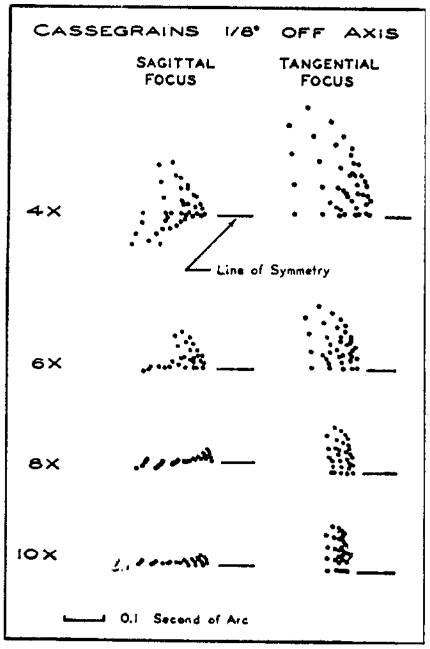


Fig. 3 (left). Drawings by Robert E. Cox of three telescope systems discussed here are reproduced to one-sixth actual scale. Their physical dimensions are given in the table on page 227 of this issue.

Fig. 4 (right). Results of the author's ray-trace calculations for off-axis images in Cassegrain systems of different secondary magnifications. The images are shown to a uniform angular scale. In all cases the f/4 primary mirror is the same. Note the increasing effect of astigmatism at higher amplifications.



portability, and decreases the effects of air currents within the tube (Fig. 3).

The **true Cassegrain** is a geometrically "pure" system, in that its parabolic primary mirror brings rays from infinity (on axis) to a perfect focus, and the hyperbolic secondary transfers them to another perfect focus. This system is free from spherical aberration. Its most serious aberration is usually coma, which Robert T. Jones[2] has shown to be the same as that of a paraboloid with the same aperture and equal in focus to the effective focal length of the Cassegrain (or Gregorian) system. Aden B. Meinel[3] points out that Cassegrain astigmatism is that of an equivalent prime-focus paraboloid, multiplied by the magnification of the secondary mirror. This can be seen in Fig. 4 by comparing the patterns for the sagittal and tangential foci at each magnification.

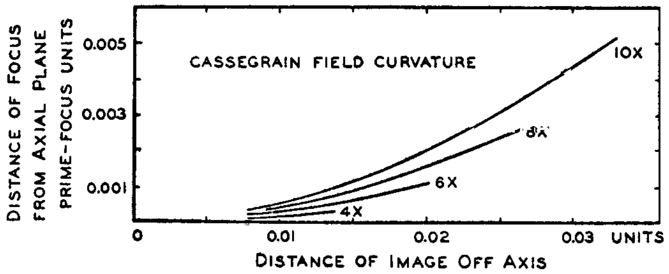
The angular resolution of these systems at a given field angle increases with secondary magnification, but this is merely the natural consequence of the increased focal ratio. At the typical Newtonian focal ratio of f/8, only 23 degrees of an orthoscopic ocular's apparent field would have visually undetectable aberrations (of less than two minutes of arc), whereas at f/12 and greater we find acceptable images over an entire 40-degree field. As for curvature of field, it also increases with the magnification (Fig. 5).

Therefore, the true Cassegrain offers performance comparable with that of a prime-focus paraboloid, but the telescope's actual length is several times less. This advantage is gained at the expense of some additional astigmatism, which is usually not detrimental, and increased curvature of field. A Gregorian with the same dimensions gives almost identical images and field curvature, except that the field is curved toward the eyepiece rather than away from it. In visual work, the eye can compensate for some of this particular kind of curvature.

The **Dall-Kirkham** form of Cassegrain has been analyzed in some detail by P. R. Yoder, Jr., F. B. Patrick, and A. E. Gee[4], and Jones has studied its coma. This system uses a convex spherical secondary (which is easier to make than a hyperbola) and an elliptical primary mirror. The undercorrected primary compensates for the spherical aberration of the secondary, which effectively transfers figuring for spherical aberration from the secondary to the primary, where it actually cancels some of the correction required. The primary can either be figured as a pure ellipse or set up with the finished secondary in an autocollimating arrangement against an optical flat and figured by a null test. There is usually no detectable difference in the results of these two methods. But the Dall-Kirkham's large coma makes its usable field of view much smaller than that of a true Cassegrain.

To avoid the adverse coma resulting from making the secondary spherical, with all the figuring on the primary, we might go the other way, using a hyperbolic primary and a more hyperbolic secondary than a true Cassegrain has. This is the **Ritchey-Chrétien** design, in which coma is essentially zero, leaving only astigmatism and curvature of field to affect resolution. At the circle of least confusion, midway between the sagittal and tangential astigmatic images, the result is less objectionable than that of an asymmetrical comatic image. The Naval Observatory's 40-inch Ritchey-Chrétien reflector at Flagstaff, Arizona, uses curved photographic plates to obtain a wider field. The 80-inch telescope at Kitt Peak is to be of this type, too.

In the past few years the **Maksutov-Cassegrain** system has been well publicized in amateur circles, particularly John Gregory's designs in Sky and Telescope (March and July, 1957, and June, 1958). These systems need no description here, except to say that first-order spherical aberration is corrected by placing in front of the secondary mirror a meniscus shell of low power and steep curvature. In Gregory's designs the secondary is actually a small aluminized spot on the back surface of the corrector and is therefore



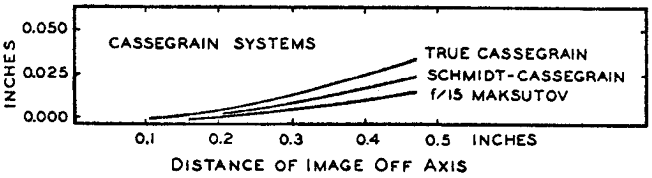


Fig. 5. At left, Cassegrain field curvature is shown in units of the primary's focal length. Above, field curvature in several systems is compared, the units being inches.

spherical, as in the Dall-Kirkham. The primary is also spherical, with all spherical-aberration correction transferred to the correcting lens. This transfer of figuring toward the center of curvature of the primary was the guiding theme for Bernhard Schmidt's invention.

The advantages of this system over the simple refractor are, of course, its short length and its greatly reduced chromatic aberration. The usual spider support in a reflector is absent, and the field curvature is smaller than that of a simple Cassegrain (Fig. 5). The closed tube has advantages, too.

Gregory-Maksutov telescopes have been designed in two sizes, f/15 and f/23. The latter seems quite satisfactory with spherical surfaces alone, but those of an f/15 cannot all be spherical. About one fringe of figuring is required on the corrector, or about one third of this amount if figured on the primary mirror. Color in Gregory's first f/15 design (July, 1957, page 440) was small but not undetectable – for C and F light the images were not diffraction-limited if D light was in focus. Gregory redesigned the f/15 to reduce the color (June, 1958, page 423), focusing C, D, and F light all within the visual disk image. Coma is found to be somewhat worse than that of a true Cassegrain of the same size and focal length, but the f/15 is satisfactory for most work.

The last and most complex member of the Cassegrain family that I have examined is the Schmidt-Cassegrain, which has its focus inside the system and requires a transfer optical arrangement for visual use. We might regard it as a Maksutov whose corrector shell and aluminized spot are replaced with a Schmidt plate and a separate secondary mirror. Compared with a special flat-field Schmidt design by James G. Baker[5], we have sacrificed some flatness of field to bring the focus outside of the system, where it can be conveniently observed visually.

The Schmidt-Cassegrain retains the following advantages of the Maksutov-Cassegrain: compactness; spherical primary mirror; possibility of secondary mirror mounted on the corrector plate to eliminate spider diffraction; less curvature of field than in the true Cassegrain; and negligible chromatic aberration.

The secondary mirror and the corrector are aspheric, with their figuring adjusted to eliminate coma. Astigmatism is undetectable. The secondary is a conic section – a convex ellipsoid that must be tested by zones, or by interference using a concave ellipsoid figured on the tool.

The corrector lens can be almost as large as the primary mirror, giving a larger clear aperture than an equivalent Maksutov-Cassegrain does. Since the corrector blank is nearly flat and need be only thick enough for structural support, much weight and glass are saved, and generation costs are less than with a Maksutov instrument. This element can be figured against the finished primary mirror, using the Foucault test with the appropriate zonal radii.

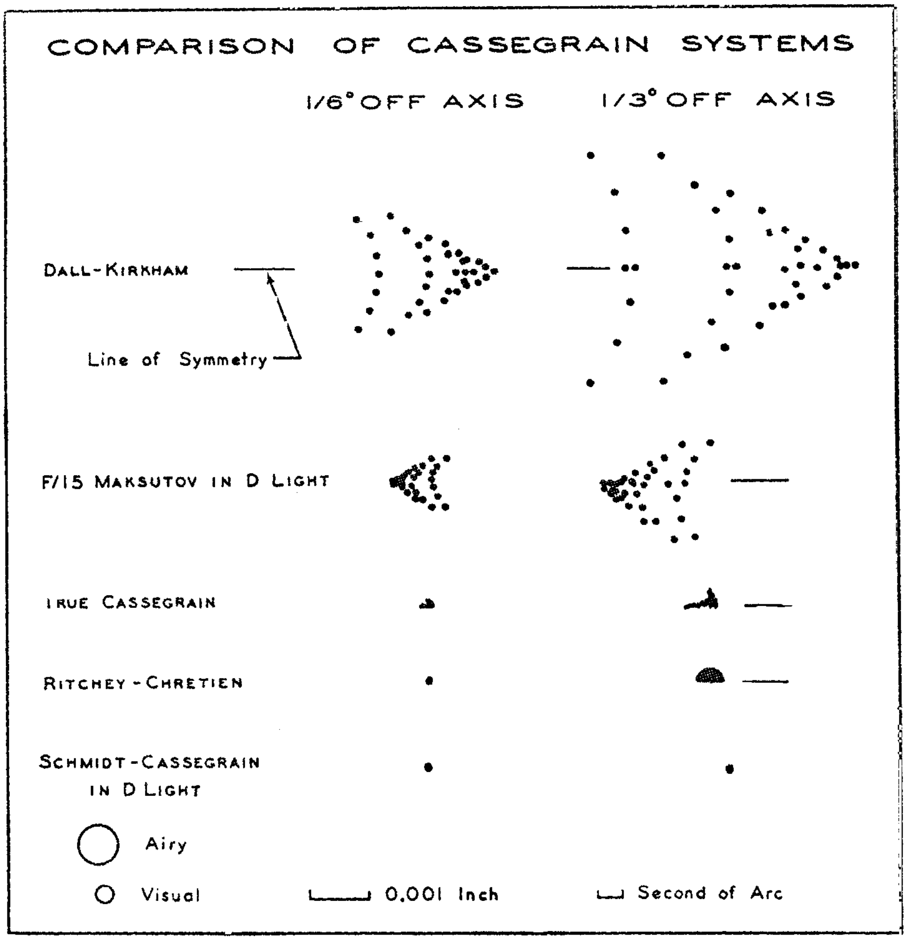


Fig. 6. Off-axis performances of five telescopes of the same dimensions. The Airy disk diameter is 0.732 mil, the visual diffraction disk 0.300 mil. The Maksutov has coma opposite in sense to that of the other systems.

Schmidt-Cassegrain construction is one step more difficult than that of a Maksutov, because an additional surface has to be made and figured. But the thickness and radii of curvature are not nearly as critical in Schmidt optics.

In Fig. 6 various systems are compared, their physical dimensions, focal ratios, and focal lengths being essentially the same as Gregory's f/15 Maksutov. These instruments are similar in every possible structural parameter and can be placed on the same size mounting; their specifications are given in Gleanings for ATM's in this issue (page 227).

Since each observer has his own requirements, too specific conclusions should not be drawn here. It is well to note that off-axis resolution is relatively unimportant in planetary observations, for which the Dall-Kirkham instrument is well suited. And in photography, the smallest point that ordinary emulsions will resolve is between 0.0003 and 0.001 inch (usually nearer the latter).

Robert E. Cox points out that many observers with small-aperture instruments may find their own results far from matching the spot diagrams. In amateur telescopes, imperfect or slightly misaligned optics, tube currents, and inferior eyepieces all tend to mask the true diffraction pattern. Astronomical seeing always impairs it, so actual compound instruments are best tested in a controlled laboratory environment.

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