

## Overcoming low index limitations in antireflection coatings with additional thickness

Ronald R. Willey

Opto Mechanik, Inc.  
P.O. Box 361907  
Melbourne, Florida 32936

### ABSTRACT

It has been shown empirically<sup>1</sup> that the lack of very low index of refraction materials is the major limitation to achieving a very low reflectance coating over a very broad spectral band. It has been shown in the same work that additional coating thickness can be employed to make up for this deficiency to a certain extent. Thicknesses which are an order of magnitude thicker than the minimum necessary for a reasonable very broad band antireflection (AR) coating can reduce the reflection to about one half that of the minimum thickness case. This result is empirically predictable to a satisfactory degree, but the underlying reasons for this have not been clear. This paper explores the principles which contribute to the understanding of this effect of additional thickness by empirical and comparative means. The Fourier viewpoint adds to the understanding of variations from the ideal design. We have previously shown<sup>2</sup> that the ideal AR coating, when any and all indices are available, would be a smooth inhomogeneous "step-down" in index from the substrate to the medium. The form of the index profile is approximately a Gaussian decay from the substrate to the medium or similarly described by Southwell<sup>3</sup> as a "quintic function". When the medium is a vacuum or air and the lowest available index is represented by a real material such as MgF<sub>2</sub> at index 1.38 rather than values very close to that of the medium, the discontinuity from the smooth step-down profile causes a reflection residual that cannot be overcome by adjustments in the rest of the smooth profile. Additional thickness and the appropriate index profile can be used to reduce, but not eliminate entirely, this residual reflection. We discuss our observations and findings on these effects in more detail in the body of this paper.

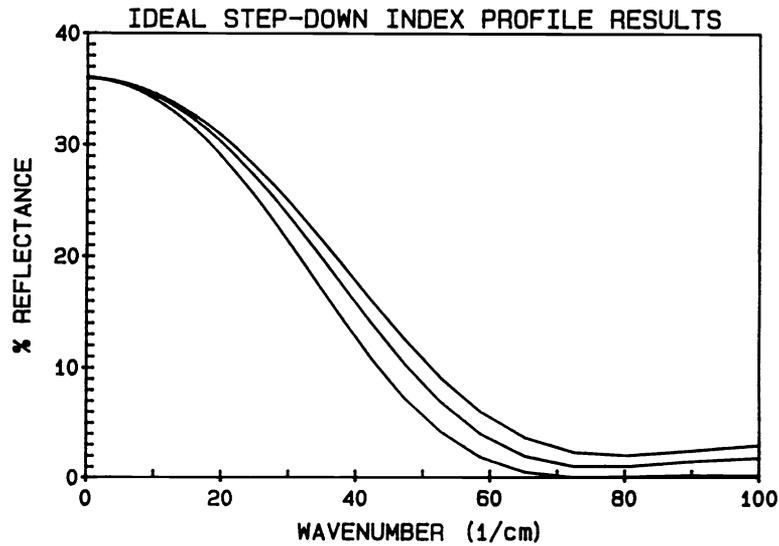
### 1. INTRODUCTION

There are three principal points in this paper concerning broadband AR coatings. The first is that the lowest index of refraction available limits the lowest reflectance achievable over a broad band. The second is that extra thickness in the film structure can reduce the reflectance in the band somewhat. The third is a discussion of why the extra thickness is of benefit. The first two points have been discussed extensively in earlier works<sup>1,2</sup>. We will review them briefly here before going on to the third point.

### 2. LOW INDEX LIMITATIONS

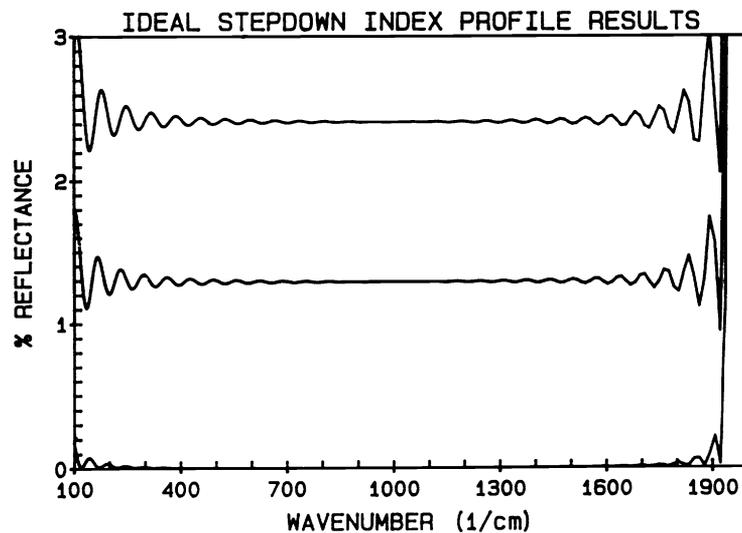
If the AR requirement were for only a narrow band such as the classical "V" coat, the lowest index available is of no consequence. It is, in fact, possible to design a coating to antireflect a single wavelength with layer materials whose indices are both higher than that of the substrate. However, as the desired bandwidth increases and the reflectance requirement decreases, the effects of the lowest index material becomes much more important. We showed<sup>1</sup> empirically that the minimum average reflectance intensity ( $R$ ) in a moderate bandwidth goes approximately as  $(L-1)^{3.5}$ , where  $L$  is the lowest index available (and used as the last layer). It can be seen that for an uncoated substrate over an infinite bandwidth the  $R$  would be just the Fresnel reflectance or  $((1-n)/(1+n))^2$  where  $n$  is the substrate index. As  $n$  approaches 1.0,  $R$  approaches  $((n-1)/2)^2$ .

In our earlier work<sup>2</sup>, we showed that the ideal AR coating for the broadest band would be a smooth "step-down" function of index from the substrate to the medium (air or vacuum, typically). Various forms of the function have been proposed<sup>2,3</sup>, but the differences are generally imperceptible; the function looks generally like a Gaussian decay from the substrate to the medium. Interestingly, the resulting spectral reflectance looks the same wherein the reflectance decreases like a Gaussian function from the Fresnel value of the uncoated substrate at zero spectral frequency to a minimum value from some frequency to infinite frequency. This is shown in the lower curve of Fig. 1 for such a fictitious coating on a germanium substrate. If the lowest index available is

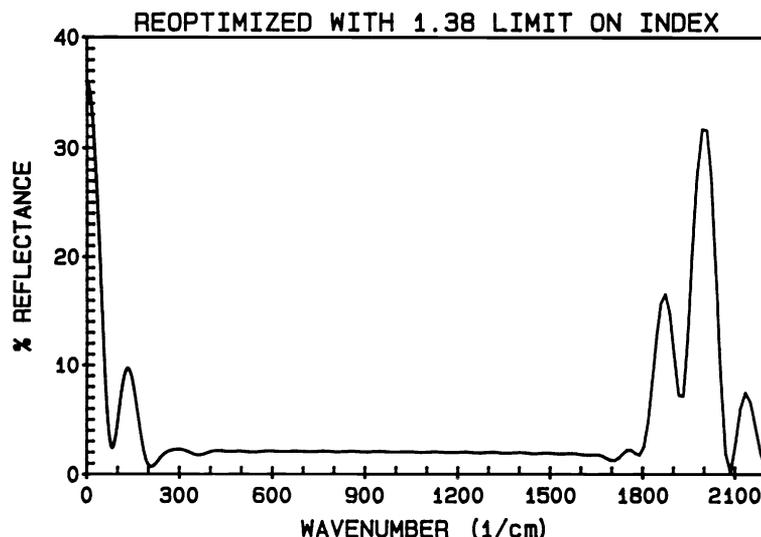


**Figure 1.** Long wavelength (low frequency) reflectance of an ideal inhomogeneous AR coating on germanium, lower curve. Middle curve for profile truncated at 1.282 index. Upper curve for 1.400 truncation.

greater than 1.0, then there will be a step discontinuity between it and the medium which causes a Fresnel reflectance. The upper curves in Fig. 1 show these effects of 1.282 and 1.400 for minimum indices. Figure 2 shows these same results over the AR band. The right end of Fig. 2 would be flat lines to infinity but for an artifice of the calculation technique. The right end is due to the fact that we had to calculate the results using a finite number of homogeneous layers (30-40) to simulate a smoothly varying homogeneous coating transition. The right end is the frequency where those finite layers become one halfwave of optical thickness. The minimum reflectance values in the band are essentially the Fresnel reflectance levels for the minimum index values. They are actually somewhat less ( $\approx 80\%$ ) which we conjecture to be caused by the fact that the simulated band is not infinite and therefore allows some reduction in the limiting reflectance.



**Figure 2.** Same as Fig. 1 over extended spectral region of the AR band.



**Figure 3.** Reoptimization of profile truncated at 1.38 index did not reduce average reflectance in the AR band but made changes outside band.

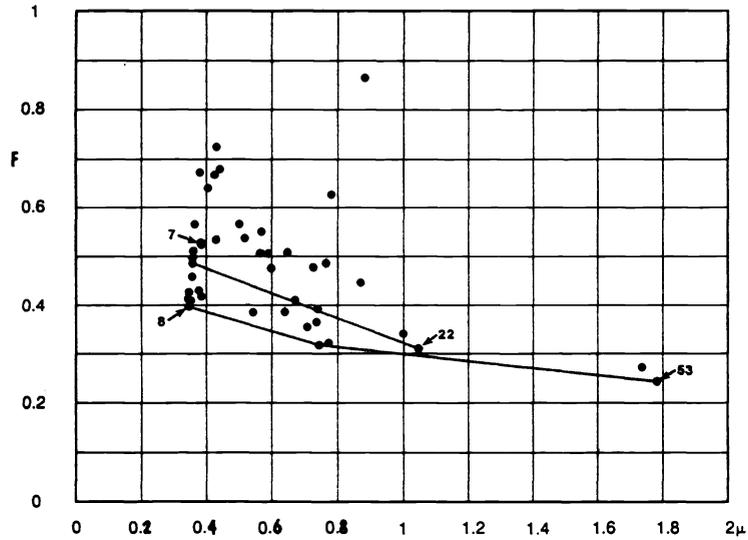
An optimization attempt was made to improve on the index profile of a smooth step-down function which was truncated at the index value of 1.38. Figure 3 shows the result which is essentially the same in the AR band as the unoptimized truncated form. We conclude that the minimum index value determines the lower limit on reflectance in a broad band and no change of the inhomogeneous profile between the substrate and the minimum index can improve the result (within the same overall thickness film). As the bandwidth is narrowed, however, it is possible to approach zero reflectance as in the limit of a "V" coat. As more thickness is added to the coating, the reflectance can also be reduced somewhat, which is what we shall discuss next.

### 3. BENEFIT OF EXTRA THICKNESS

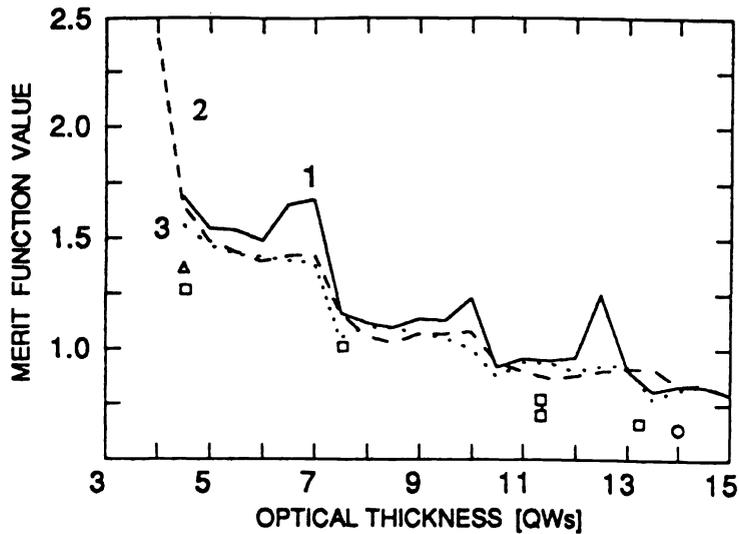
We reported<sup>1</sup> some observations from the results of an AR design contest run by Thelen and Langfeld<sup>4</sup> for the Berlin meeting in September of 1992. A figure from that report is reproduced here for convenience as Fig. 4. These are the resulting merit (a function of average reflectance in the AR band) of dozens of designs from around the world. The best designs (lowest F) agree with the prediction formula reported<sup>1</sup> and show that the average reflectance in the band decreases asymptotically to zero with thickness. This is disappointing, however, in that an order of magnitude increase in thickness is needed to cut the reflectance in half.

The same phenomenon of reflectance versus thickness can be observed in a plot arrived at from different approaches. Verly, et al.<sup>5</sup> refined and extended the work of reference 2 and we reproduce a figure from that result for the convenience of the reader as Fig. 5. It shows the decrease in reflectance (merit function) with increasing film thickness for various designs and optimization techniques. This confirms from yet a third independent set of observations that AR performance can be improved by thickness, but not very efficiently. Our estimation<sup>1</sup> is that the average reflectance in the band reduces as  $T^{-0.31}$  where T is the optical thickness in full waves at the wavelength of the geometric mean of the two extremes of the AR band. This is consistent with Figs. 4 and 5.

It also becomes apparent that there is a minimum thickness needed of approximately one wave of optical thickness at the above mentioned geometric mean wavelength for best results. This is consistent with the classical QHQ three layer AR coating which is one wave of optical thickness at 530nm for a band from 400 to 700nm.



**Figure 4.** Thelen-Langfeld<sup>4</sup> results of merit function (average reflectance in AR band) versus physical thickness from many contributions to the Berlin 1992 design problem. The number of layers is shown for selected designs. The upper line connects the best designs that did not use any titania while the lower line connects those that did. The physical thickness of four QWOTs at midband is approx.  $0.35\mu$ .

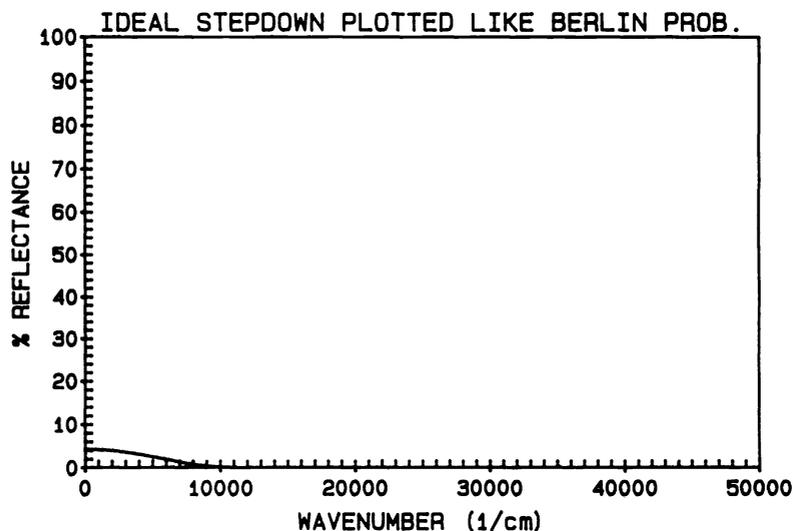


**Figure 5.** Variation of the merit function values with thickness in the presence of refractive-index constraints. The curves correspond to graded-index solutions obtained with different starting designs:  $\square$  and  $O$ , multilayer solutions;  $\Delta$ , optimized graded-index design.

It is clear that there is benefit to be gained by additional thickness beyond the minimum, but why this is true has not been well understood. We will next discuss some concepts which should aid in understanding the underlying principles.

#### 4. OBSERVATIONS OF EXTRA THICKNESS CHARACTERISTICS

Figure 6 uses the same design as the lower curve in Fig. 1, but it is scaled and plotted to match the general band for the Berlin design problem of 400-900nm ( $11111$ - $25000$   $\text{cm}^{-1}$ ). This represents the ideal step-down index profile of index from the substrate at 1.52 to the medium at 1.00. If any and all indices were available between these extremes, the coating could be "perfect" from the low frequency limit at  $11111$   $\text{cm}^{-1}$  to infinite frequency. It is interesting to note that the low frequency end of the spectral frequency plot looks similar to the shape of the index versus optical thickness profile. Figure 6 represents the best that could be done if any indices could be used; the rest of the figures will show what can be achieved when one is limited to real materials.

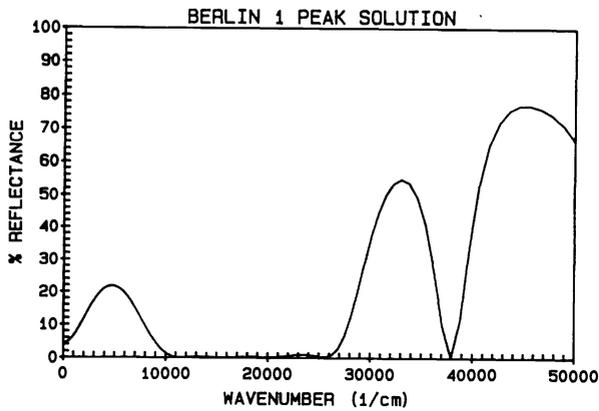


**Figure 6.** Ideal step-down inhomogeneous index profile for broadest band AR from the longwave end (900nm) of the Berlin problem to infinite frequencies. Scaled from the lower curve in Fig. 1.

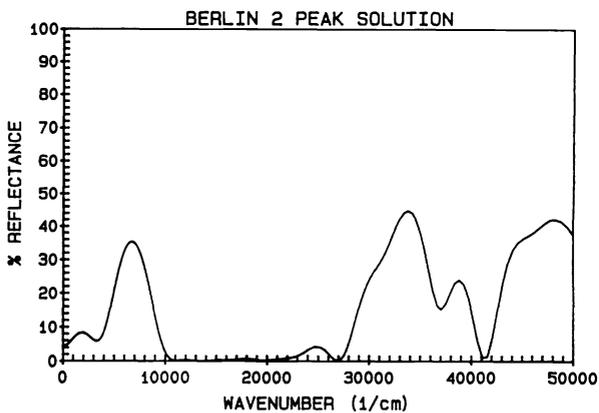
Figures 7 through 10 are adapted from real designs resulting from the Berlin problem submissions. Figure 7 is the best of the minimum thickness designs and is the 8-layer point in Fig. 4. Figure 8 is the best of the designs that are twice the minimum thickness and seen at about  $0.73\mu$  on Fig. 4. Figure 9 is the best of the designs that are three times the minimum thickness and identified as the 22-layer design in Fig. 4. Figure 10 is a design similar to the best of the submissions at five times the minimum thickness (53-layers), but it has been redesigned to 32-layers. This redesign was the result of the earlier studies<sup>1</sup> where we showed that the best designs tend to have a minimum number of layers dictated by the approximation that the number of layers should be  $6T+2$  (in the visible) where  $T$  is the minimum thickness. Note that the 8-layer design of Fig. 7 fits this and the 22-layer design of Fig. 9 is close to the 20 predicted by the formula.

Notice that the number of maxima and minima (periods) to the left of the AR band in each of Figs. 6 to 10 is equal to the number of times that the coating is thicker than the minimum ( $T$ ). From our previous reports<sup>1,2</sup>, it can also be seen that the profiles index versus thickness have the same number of extrema. In this area to the left (low frequency/long wavelength) side of the AR band are the "universal" shapes from which we hope to gain some understanding. These shapes will be generally the same for any coating designs in the same classification. The right sides of the figures, on the other hand, will generally be unique to the specific details of an individual design.

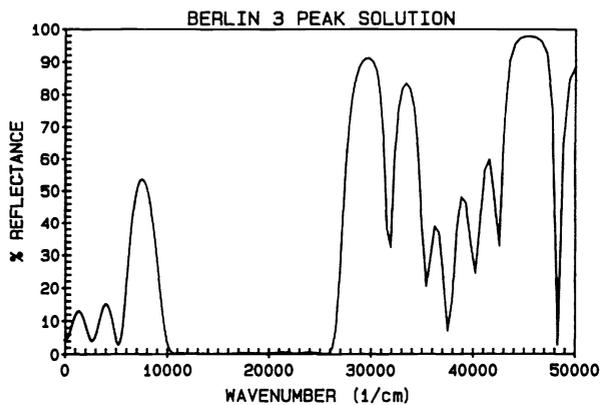
In comparing Figs. 7 to 10, it can be seen that the reflectance peak to the left of the AR band increases with increasing optical thickness of the design. In effect, the AR is a short wave pass filter (SWP) where heavy emphasis is placed on the antireflection properties for the passband. The ripples to the left of the peak are also just like those on the non-antireflected side of any QWOT-stack blocker used for SWP.



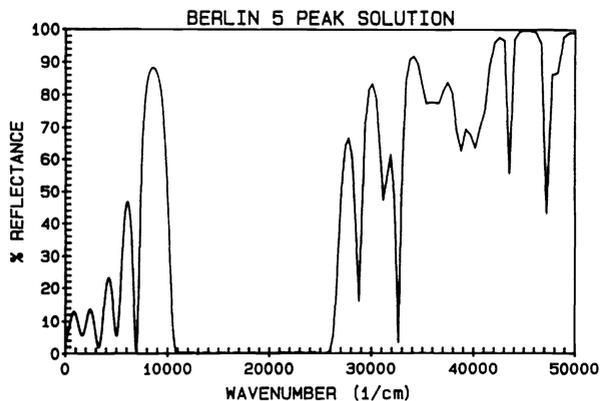
**Figure 7.** Best minimal thickness design AR from the Berlin problem (11111- 25000  $\text{cm}^{-1}$ ). Design is 1T thick and has 8 layers.



**Figure 8.** Best of Berlin designs at 2x minimum optical thickness (2T). The merit of this design is seen at the inflection point at  $0.73\mu$  of Fig. 4. The ripple in the 11111-25000  $\text{cm}^{-1}$  AR band is not there in the true design, but is an artifact of ignoring dispersion in these figures.



**Figure 9.** Best of the Berlin designs that are 3x the minimum optical thickness (3T); identified as 22-layer on Fig. 4.



**Figure 10.** Design similar to the very best of the Berlin submissions at 5T and 53-layers. This is a redesign reducing the number of layers to 32 per the prediction of our previous studies<sup>1</sup>.

It would appear that the reduction in reflection in an AR band, when real homogeneous materials are used, is accompanied by an increase in reflection outside of the band. The addition of more layers and therefore more interfaces will add reflectance over the infinite band. Therefore, the additional layers are increasing the reflectance in areas of little concern while reducing it in the AR band. We will next consider this result from the Fourier viewpoint.

## 5. FOURIER VIEW OF THE CHARACTERISTICS

If we think in Fourier terms or the frequency domain, we know that a single interface between two media of different indices constitutes a single delta function in the reflectance amplitude  $r$  versus optical thickness. Here  $r = (n_1 - n_2)/(n_1 + n_2) = \Delta n/n_{ave}$  or the Fresnel amplitude reflectance of the interface. This spike is like a lightning bolt in time that generates some of all time frequencies. In the reflectance versus thickness case, the spike generates a reflectance which is equal over all spectral frequencies. If we have two interfaces as in a single layer AR coating, two spikes are generated. These interfere with each other to produce a single sinusoidal function whose period depends on the spacing of the pulses or the optical thickness of the layer. This simplest form of AR coating can reduce the reflectance over a narrow band which is equal to one period. If two layers are involved, there are three interfaces which generate three frequencies by each of three interfaces taken two at a time. Three layers have four interfaces which generate a total of six frequencies. Generally,  $L$  layers will generate  $(L+1)!/2(L-1)!$  frequencies. If all of the interfaces were between the same materials such as  $TiO_2$  and  $SiO_2$ , each frequency would have the same amplitude and differ only in phase. For example, if all the layers were of equal thickness as in a QWOT stack, all the frequencies would be in phase and add for maximum reflectance at the QWOT wavelength. The highest frequency will be generated by the two interfaces which are most widely separated in optical thickness. This points to the fact that additional thickness in an AR coating provides higher frequencies which can be used to reduce residual reflectance in the AR band. The optimization process basically is adjusting the phases of the various frequency components to achieve the desired reflectance results as closely as possible within the constraints.

We previously<sup>1</sup> found that the number of ripples in an AR band were approximately  $8B/3 + 2T - 4$ . Here  $B$  is the ratio of the longest wavelength to the shortest wavelength and  $T$  is the optical thickness as defined above. This observation shows that the ripples are a linear function of bandwidth and thickness. If we increase the thickness while keeping the  $B$  constant, we will increase the number of ripples (higher frequencies). This is consistent with concepts put forth above.

The whole process can be related to the Fourier synthesis of a square waveform by adding together appropriate amplitudes and phases of the sinusoidal harmonics of the fundamental frequency of the square wave. As more harmonics are added, the ripples reduce in amplitude and increase in frequency until the result is not distinguishable from a true square wave. This seems to be the same case for a broad band AR coating design. There is also the possibility of applying this same concept to the design of broadband high reflectors.

## 6. CONCLUSIONS

We have shown empirically that the lowest available index limits what can be achieved in a broadband AR coating design because the residual discontinuity between it and the medium produces a Fresnel reflectance similar to that of an uncoated interface of the same material. The benefits of additional thickness in the coating beyond the minimum for a good AR have been shown from the author's work,<sup>1</sup> Fourier design,<sup>2</sup> and a collection of many designers' work.<sup>4</sup> We have discussed some viewpoints that should be of help in understanding the phenomena by which additional thickness improves a broadband AR coating and why it quickly reaches a point of diminishing returns.

## 7. REFERENCES

1. R. R. Willey, "Predicting achievable design performance of broadband antireflection coatings," *Appl. Opt.* 32, 5447-5451 (1993).
2. R. R. Willey, P. G. Verly, J. A. Dobrowolski, "Design of wideband antireflection coatings with the Fourier transform method," in Optical Thin Film Applications, R. Herrmann, ed., SPIE 1270, 36-44 (1990).
3. W. H. Southwell, "Gradient index antireflection coatings," *Opt. Lett.* 8, 584 (1983).
4. A. Thelen and R. Langfeld, "Coating design problem," in Thin Films for Optical Systems, K. H. Guenther, ed., SPIE 1782 (1992).
5. P. G. Verly, J. A. Dobrowolski, R. R. Willey, "Fourier-transform method for the design of wideband antireflection coatings," *Appl. Opt.* 31, 3836-3846 (1992).